**MTH 205**
**Final Exam Review**

1. State the <u>order</u> of each equation and whether the equation is <u>linear or nonlinear</u>.

   (a) $y''' + xy'' = \sin x$

   (b) $\dfrac{dy}{dx} + xy = \tan x$

   (c) $\dfrac{d^2 y}{dx^2} + y^2 = e^x$

   ANS: (a) third order, linear
   (b) first order, linear
   (c) second order, nonlinear

2. Separate variables and solve $dy + x(y+1)\,dx = 0$.

   ANS: $y = ce^{-x^2/2} - 1$

3. Make use of homogeneous functions to solve $2xydx + (x^2 + y^2)\,dy = 0$.

   ANS: $3x^2 y + y^3 = c$

4. Consider the ODE $(x - 2xy + e^y)\,dx + (y - x^2 + xe^y)\,dy = 0$.
   (a) Show that the equation is exact.
   (b) Solve the problem above with the initial condition $y(1) = 0$.

   ANS: (a) $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} = -2x + e^y$
   (b) $x^2 - 2x^2 y + 2xe^y + y^2 = 3$

5. Find the general solution to $y' - 2xy = e^{x^2}$.

   ANS: $y = e^{x^2}(x + c)$

6. Consider the IVP $y' = 1 - t + 4y$, $y(0) = 1$. Use Euler's method to approximate $y(0.2)$ using a stepsize of $h = 0.1$.

   ANS: $y(0.2) \approx 2.19$

7. A body whose temperature is $100^{\circ}$ is placed in a medium which is kept at a constant temperature of $20^{\circ}$. In ten minutes the temperature of the body falls to $60^{\circ}$.
   (a) Find the temperature *T* of the body as a function of time *t*.
   (b) Find the temperature of the body after 40 minutes.
   (c) When will the body's temperature be $50^{\circ}$?

   ANS: (a) $T(t)=20\left(1+4e^{-0.06931t}\right)$
   (b) $25^{\circ}$
   (c) 14.2 min

8. Use the method of exact differentials to solve $\left(y^2+y\right)dx-xdy=0$.

   ANS: $x+\dfrac{x}{y}=c$

9. Show that the differential equation $\left(e^x-\sin y\right)dx+\cos y dy=0$ is not exact and then find an integrating factor.

   ANS: $\dfrac{\partial M}{\partial y}=-\cos y,\quad \dfrac{\partial N}{\partial x}=0,\quad \text{Integrating factor}=e^{-x}$

10. Use a substitution to solve the ODE $\left(2x+3y-1\right)dx+\left(4x+6y+2\right)dy=0$.

    ANS: $2x+3y-1-8\ln\left|2x+3y+7\right|+y=c$

11. Solve the Bernoulli differential equation $y'+xy=\dfrac{x}{y^3}$.

    ANS: $y^4=1+ce^{-2x^2}$

12. Solve $y'+2x=2\left(x^2+y-1\right)^{2/3}$. Hint: Let $u=x^2+y-1$.

    ANS: $y=1-x^2+\dfrac{\left(2x+c\right)^3}{27}$

13. Determine whether the functions are linearly independent or linearly dependent.
    (a) $t^2+5t,\ t^2-5t$
    (b) $e^x,\ e^{2x},\ e^{3x}$
    (c) $x,\ x^{-1}$

ANS: (a) independent
(b) independent
(c) independent

14. Solve the IVP $y'' - 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$.

ANS: $y = 2e^{x} - e^{2x}$

15. Find the solution to $\left(D^4 - 3D^2 + 2D\right) y = 0$.

ANS: $y = c_1 + \left(c_2 + c_3 x\right) e^{x} + c_4 e^{-2x}$

16. Find the solution to $y'' - 2y' + 5y = 0$.

ANS: $y = e^{x}\left(c_1 \cos 2x + c_2 \sin 2x\right)$

17. Use the method of undetermined coefficients to solve
(a) $y'' - 3y' + 2y = 2xe^{3x} + 3\sin x$
(b) $y'' + 4y' + 4y = 3e^{-2x}$

ANS: (a) $y = c_1 e^{x} + c_2 e^{2x} + xe^{3x} - \frac{3}{2} e^{3x} + \frac{3}{10} \sin x + \frac{9}{10} \cos x$
(b) $y = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{3}{2} x^2 e^{-2x}$

18. Use variation of parameters to find the solution to $y'' + y = \tan x$.

ANS: $y = c_1 \cos x + c_2 \sin x - \cos x \ln\left|\sec x + \tan x\right|$

19. Find the general solution of $x^2 y'' + xy' - y = 0$ given that $y = x$ satisfies the differential equation. Hint: Use reduction of order.

ANS: $y = c_1 x + c_2 x^{-1}$

20. Show, by integration, that $L\{t\} = \frac{1}{s^2}$.

ANS: Evaluate $L\{t\} = \int_0^\infty te^{-st}\,dt$ by integration by parts.

21. Use the method of Laplace transforms to solve $y'' + 2y' + y = 1$ for which $y(0) = 2$ and $y'(0) = -2$.

ANS: $y = 1 + e^{-x} - xe^{-x}$

22. Compute $L^{-1}\left\{\dfrac{2s+1}{s^2 - 2s + 2}\right\}$

ANS: $2e^t \cos t + 3e^t \sin t$