Math 205 Review

Separable equations: $\frac{dy}{dx} = \frac{g(x)}{h(y)}$

Examples: 1) $(1+x)\,dy - y\,dx = 0$. $y=c(1+x)$

2) $xy^4\,dx + (y^2 + 2)e^{-3x}\,dy = 0$. $9y^{-1}+6y^{-3}=e^{3x}(3x-1)+c$

Exact equations: $M(x,y)\,dx + N(x,y)\,dy = 0$ where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Example: 1) $2xy\,dx + (x^2 - 1)dy = 0$. $x^2y-y=c$

Linear: $\frac{dy}{dx} + P(x)y = f(x)$: **Integrating factor** $\mu(x) = e^{\int p(x)dx}$, $y = \frac{1}{\mu(x)}\left[\int \mu(x) f(x)dx + C\right]$

Example: 1) $dy/dx + 2xy = x$ subject to $y(0) = -3$ $y = \frac{1}{2} - \frac{7}{2}e^{-x^2}$

Reduction of Order: Given a solution f(x) to y'' + py' + qy = 0, a second linearly independent solution is given by $y = f(x)\int \frac{e^{-\int p(x)dx}}{f(x)^2}\,dx$

Example: 1) Given $y = x^3$ is a solution to $x^2y'' - 6y = 0$, find a second solution. $y=x^{-2}$

Bernoulli Equations

Homogeneous Linear with constant coefficients: $a_ny^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_2y'' + a_1y' + a_0y = 0$

Using $y = e^{rx}$ yields 3 possibilities for roots in auxiliary equation

Case I) real different roots

$Y = c_1e^{r_1x} + c_2e^{r_2x} + \cdots$

Case II) complex roots $a \pm bi$

$Y = e^{ax}(c_1\cos bx + c_2\sin bx)$

Case III) repeated roots

For each duplicate root, multiply first solution by Successive powers of x to form linearly independent solutions.

Nonhomogeneous Linear with constant coefficients: The Method of Undetermined Coefficients.

$a_ny^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_2y'' + a_1y' + a_0y = g(x)$

Example 1) $y'' - 3y' = 8e^{3x} + 4\sin x$ solution: $y = c_1 + c_2e^{3x} + \frac{8}{3}xe^{3x} + \frac{6}{5}\cos x - \frac{2}{5}\sin x$

Variation of Parameters: $y'' + Py' + Qy = f(x)$.

Given y_1 and y_2 form a fundamental solution set when $f(x) = 0$,

$$u_1' = -\frac{y_2 f(x)}{W}, \quad u_2' = \frac{y_1 f(x)}{W}, \quad y_p = u_1 y_1 + u_2 y_2$$

Example: y'' + y = sec x y = $c_1$cos x + $c_2$sin x +x sin x +cos x ln|cos x|

Variation of Parameters: Higher order linear, nonhomogeneous with constant coefficients.

Use $y_p = \sum_{k=1}^{n} y_k(x) \int \frac{g(x) W_k}{W}\, dx$ to solve $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_2 y'' + a_1 y' + a_0 y = g(x)$

where W_k are the corresponding Wronskians.

Mass Spring Problems: 5.1-5.3

$$mx'' + \beta x' + kx = g(t),\ x(0) = x_0, x'(0) = x'_0$$

Cauchy-Euler: $ax^2y'' + bxy' + cy = f(x)$ using the substitution $x = e^t$, transforms the equation to $ay(t)'' + (b-a)y(t)' + cy(t) = f(e^t)$

Example: $x^2y'' + xy' + 4y = 0$, x>0. $c_1\cos(2 \ln x) + c_2\sin(2 \ln x)$

Or assuming the form $y = x^m$, we look at different cases for the roots of the auxiliary equation.

Series Solutions: Solving linear, constant and variable(analytic) coefficient differential equations using power series expansions about a specified value of x (around ordinary points only) 6.3

Example. Solve $y' = y^2$ (nonlinear) using a series expansion about x = 0. $y = \frac{1}{1-x}$

Method of Laplace Transform: Finding transforms and inverse transforms using definition as well as theorems. Solving linear, constant coefficient initial value problems. 7.1-7.5

Direction Fields: Finding isoclines and the fields for dy/dx = f(x,y)

Orthogonal Trajectories: Given dy/dx = f(x,y), $\frac{dy}{dx} = \frac{-1}{f(x,y)}$ is the diff. equation of orthogonal family

Approximation Methods: Euler and Improved Euler.