Review 2

1. $W = \begin{vmatrix} 4x & \sin x & \cos x \\ 4 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \end{vmatrix}$

$= 4x\left[-\cos^2 x - \sin^2 x\right] - 4\left[-\sin x\cos x + \sin x\cos x\right] + 0$

$= -4x$ NOT identically zero $\therefore$ Linearly independent

2. let $c_1 = -1$, $c_2 = 1$, $c_3 = 1$

$c_1 \cdot 1 + c_2\cos^2 x + c_3\sin^2 x$

$= -1 + \cos^2 x + \sin^2 x$

$= -1 + 1$

$= 0$ $\therefore$ Linearly dependent

3. let $c_1 = 1$, $c_2 = 0$, $c_3 = 0$

$c_1 \cdot 0 + c_2 \cdot x + c_3 x^3$

$= 0 \cdot 1 + 0x + 0x^3$

$= 0$ $\therefore$ Linearly dependent

4. $\begin{vmatrix} 2x & e^{-x} & e^{x} \\ 2 & -e^{-x} & e^{x} \\ 0 & e^{-x} & e^{x} \end{vmatrix} = 2x\left[-1-1\right] - 2\left[1-1\right] + 0$

$= -2x$ (not identically 0, so linearly independent)

$y = x^2, \quad x^2 y'' + 2xy' - 6y = 0$

$$y'' + \frac{2}{x}y' - \frac{6}{x^2}y = 0 \qquad x \neq 0$$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

$$= x^2 \int \frac{e^{-\int \frac{2}{x}dx}}{x^4} dx$$

$$= x^2 \int \frac{e^{-2\ln x}}{x^4} dx$$

$$= x^2 \int \frac{x^{-2}}{x^4} dx$$

$$= x^2 \int x^{-6} dx$$

$$= x^2 \left(\frac{-x^{-5}}{5} + c \right)$$

$$= \frac{-x^{-3}}{5} + cx^2$$

letting $c = 0$, $y_2 = x^{-3}$ (the simplest constant multiple of $-\frac{x^{-3}}{5}$)

#6 $y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$

$m^2 - 2m + 2 = 0$

$m = 1 \pm i$

$y_c = e^x(c_1 \cos x + c_2 \sin x)$

$y_p = e^{2x}(A\cos x + B\sin x)$

$y_p' = e^{2x}((2A+B)\cos x + (2B-A)\sin x)$

$y_p'' = e^{2x}((3A+4B)\cos x + (3B-4A)\sin x)$

substituting gives

$A + 2B = 1, \quad B - 2A = -3$

$A = \frac{7}{5}, \; B = -\frac{1}{5}$

$y = e^x(c_1\cos x + c_2 \sin x) + e^{2x}\left(\frac{7}{5}\cos x - \frac{1}{5}\sin x\right)$

7. $y'' + 4y = (x^2-3)\sin 2x$

a. $m^2 + 4 = 0$

$m = \pm 2i$

$y_c = c_1\cos 2x + c_2 \sin 2x$

b. $y_p = (Ax^3 + Bx^2 + Cx)\sin 2x + (Dx^3 + Ex^2 + Fx)\cos 2x$

c. This is Long. $A=0,\ B=\frac{1}{16},\ C=0,\ D=\frac{-1}{12},\ E=0,\ F=\frac{25}{32}$

$y_p = \frac{1}{16}x^2\sin 2x + \left(-\frac{1}{12}x^3 + \frac{25}{32}x\right)\cos 2x$

8. $y'' - 2y' + y = e^x$

$y = e^{mx}$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1$

$y_c = c_1e^x + c_2xe^x$

$y_p = Ax^2e^x$

$y_p' = 2Axe^x + Ax^2e^x$

$y_p'' = 2Ae^x + 2Axe^x + 2Axe^x + Ax^2e^x$

$\cancel{Ax^2e^x} + \cancel{4Axe^x} + 2Ae^x - \cancel{4Axe^x} - \cancel{2Ax^2e^x} + \cancel{Ax^2e^x} = e^x$

$2A = 1$

$A = \frac{1}{2}$

$y = c_1e^x + c_2xe^x + \frac{1}{2}x^2e^x$

9. $y'' - y = x + \sin x$

$m^2 - 1 = 0$

$m = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$y = Ax + B$

$y' = A$

$y'' = 0$

$-Ax - B = x$

$A = -1, B = 0$

$y = -x$

$y = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$-2A\cos x - 2B\sin x = \sin x$

$A = 0, -2B = 1$

$B = -\frac{1}{2}$

$y = c_1 e^{x} + c_2 e^{-x} - x - \frac{1}{2}\sin x$

9. $y'' - y = x + \sin x$

$m^2 - 1 = 0$

$m = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$y = Ax + B$

$y' = A$

$y'' = 0$

$-Ax - B = x$

$A = -1, B = 0$

$y = -x$

$y = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$-2A\cos x - 2B\sin x = \sin x$

$A = 0, -2B = 1$

$B = -\frac{1}{2}$

$y = c_1 e^{x} + c_2 e^{-x} - x - \frac{1}{2}\sin x$

#10 $y'' - 2y' + 2y = e^x \tan x$

$m^2 - 2m + 2 = 0$

$m = \frac{2 \pm \sqrt{4-8}}{2}$

$= \frac{2 \pm 2i}{2}$

$= 1 \pm i$

$y_c = e^x (c_1 \cos x + c_2 \sin x)$

$y_p = u_1 \cos x \cdot e^x + u_2 \sin x \cdot e^x$

$$W = \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x \cos x - e^x \sin x & e^x \sin x + e^x \cos x \end{vmatrix}$$

$= e^{2x} \cos x \sin x + e^{2x} \cos^2 x - e^{2x} \sin x \cos x + e^{2x} \sin^2 x$

$= e^{2x}$

$$W_1 = \begin{vmatrix} 0 & e^x \sin x \\ e^x \tan x & e^x \sin x + e^x \cos x \end{vmatrix} = -e^{2x} \frac{\sin^2 x}{\cos x}$$

$$W_2 = \begin{vmatrix} e^x \cos x & 0 \\ e^x \cos x - e^x \sin x & e^x \tan x \end{vmatrix} = e^{2x} \sin x$$

$$u_1' = \frac{W_1}{W}$$

$$= \frac{-e^{2x}\sin^2 x}{e^{2x}\cos x}$$

$$= \frac{-\sin^2 x}{\cos x}$$

$$u_1 = \int \frac{-\sin^2 x}{\cos x}\,dx$$

$$= \int \frac{\cos^2 x - 1}{\cos x}\,dx$$

$$= \int (\cos x - \sec x)\,dx$$

$$= \sin x - \ln|\sec x + \tan x|$$

$$u_2' = \frac{e^{2x}\sin x}{e^{2x}}$$

$$u_2 = \int \sin x\,dx$$

$$= -\cos x$$

$$y_p = e^x\cos x\left(+\sin x - \ln|\sec x + \tan x|\right) - e^x\sin x\cos x.$$

$$= -e^x\cos x\ln|\sec x + \tan x|$$

$$y = e^x(c_1\cos x + c_2\sin x) - e^x\cos x\ln|\sec x + \tan x|$$

11. $\frac{1}{4}x'' + x' + x = 0, \quad x(0) = 4, \; x'(0) = 2$

$x'' + 4x' + 4x = 0$

$m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2$

$x = c_1 e^{-2t} + c_2 t e^{-2t}$

$x(0) = 4 = c_1$

$x' = -8e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$x'(0) = 2 = -8 + c_2$

$c_2 = 10$

$x = 4e^{-2t} + 10te^{-2t}$

12. Answers vary. The transient part becomes negligible as t increases.

14. graphs need to be supplied to answer this question.