EXERCISES 2.1 ; 2.2 ZILL

2.1 #1 $\frac{dy}{dx} = y^{2/3}$

$$\frac{\partial f}{\partial y} = \frac{3}{2} y^{-1/3}$$

This equation will have a solution on any Region where $y \neq 0$.

2.1 # 3. $x \frac{dy}{dx} = y$

$$\left.\begin{array}{l} \frac{dy}{dx} = \frac{y}{x} \\ \frac{\partial f}{\partial y} = \frac{1}{x} \end{array}\right\}$$

Continuous for $x \neq 0$
Unique solution Here.

2.1 # 5. $(4 - y^2) y' = x^2$

$$\frac{dy}{dx} = \frac{x^2}{4-y^2} = x^2(4-y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = 2y(4-y^2)^{-2} x^2$$

$$= \frac{2x^2 y}{(4-y^2)^2}$$

There will be a Unique Solution For R
(Both continuous for $y \neq \pm 2$.)
where $y \neq \pm 2$.

2.1 #11 $y' = 3y^{2/3}$, $y(0) = 0$

Find a solution by inspection.

$y = 0$ is the simplest.

$y = x^3$ is another

2.1 #13 $y' = y^3$; $y(0) = 0$. Find a solution. Is it unique?

$\frac{\partial f}{\partial y} = 3y^2$ f and $\frac{\partial f}{\partial y}$ are continuous in xy plane so there is a unique solution.

Simplest would be $y = 0$

$\frac{dy}{dx} = y^3$

$\frac{dy}{y^3} = dx$

$\frac{-y^{-2}}{2} = x + c$

$\frac{-1}{2y^2} = x + c$

~~$-1 = 2xy^2 + y$~~

$-2y^2 = \frac{1}{x+c}$ ?

$y^2 = \frac{-1}{2(x+c)}$

mead

2.2

#1 $\frac{dy}{dx} = \sin 5x$

$dy = \sin 5x \, dx$

$y = -\frac{1}{5}\cos 5x + C$

#3. $dx + e^{3x} dy = 0$

$e^{-3x} dx = -dy$

$-\frac{1}{3}e^{-3x} = -y + c$

$y = \frac{1}{3}e^{-3x} + c$

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2.2

#7 $xy' = 4y$

$x\frac{dy}{dx} = 4y$

$\frac{dy}{y} = \frac{4\,dx}{x}$

$\ln|y| = 4\ln|x| + 4\ln c$

$\boxed{y = c_1x^4}$

#9 $\frac{dy}{dx} = \frac{y^3}{x^2}$

$\frac{dy}{y^3} = \frac{dx}{x^2}$

$\frac{y^{-2}}{-2} = \frac{x^{-1}}{-1} + c$

$\boxed{\frac{1}{y^2} = \frac{2}{x} + c_1}$

$x = 2y^2 + c_1xy^2$

#13 $\frac{dy}{dx} = e^{3x+2y}$

$\frac{dy}{e^{2y}} = e^{3x}dx$

$e^{-2y}dy = e^{3x}dx$

$-\frac{e^{-2y}}{2} = \frac{1}{3}e^{3x} + c$

$2e^{3x} + 3e^{-2y} = c_1$

#23 $\frac{dP}{dt} = P - P^2$

$\frac{dP}{P - P^2} = dt$

$\frac{1}{P(1-P)} = \frac{A}{P} + \frac{B}{1-P}$

$1 = A(1-P) + PB$

$P = 1 \Rightarrow 1 = B$

$P = 0 \Rightarrow 1 = A$

$\left(\frac{1}{P} + \frac{1}{1-P}\right)dP = dt$

$\ln|P| - \ln|1-P| = t + c$

$\ln\left|\frac{P}{1-P}\right| = t + c$

$e^{t+c} = \frac{P}{1-P}$

$c_1e^t = \frac{P}{1-P}$

$(1-P)c_1e^t = P$

$c_1e^t - Pc_1e^t = P$

$c_1e^t = P + Pc_1e^t$

$c_1e^t = P(1 + c_1e^t)$

$P = \frac{c_1e^t}{1+c_1e^t}$

Mead

2.2

\#19 $y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$

$x^2 \ln x \, dx = \frac{(y+1)^2}{y} dy$

$\frac{x^3 \ln x}{3} - \frac{x^3}{9} = \int (y + 2 + \frac{1}{y}) dy$

$\frac{x^3 \ln x}{3} - \frac{x^3}{9} = \ln|y| + 2y + \frac{y^2}{2} + c$

$u = \ln x \quad dv = x^2$

$du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$

$\frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$

$\frac{x^3 \ln x}{3} - \frac{x^3}{9}$

\#25 $\sec^2 x \, dy + \csc y \, dx = 0$

$\frac{dy}{\csc y} = -\frac{dx}{\sec^2 x}$

$\sin y \, dy = -\cos^2 x \, dx$

$-\cos y = -\int \frac{1 + \cos 2x}{2} dx$

$-\cos y = -\frac{x}{2} - \frac{\sin 2x}{4} + c$

$\cos y = \frac{x}{2} + \frac{\sin 2x}{4} + c_1$

$4 \cos y = 2x + \sin 2x + C_2$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos 2x = \cos^2 x - \sin$

$= 1 + \cos 2x -$

\#27 $e^y \sin 2x \, dx + \cos x (e^{2y} - y) dy = 0$

$\frac{\sin 2x \, dx}{\cos x} = e^{-y}(y - e^{2y}) dy$

$\frac{\sin 2x}{\cos x} dx = (y e^{-y} - e^{y}) dy$

$\int \frac{2 \sin x \cos x \, dx}{\cos x} = \int y e^{-y} dy - \int e^{y} dy$

$-2 \cos x = -y e^{-y} - e^{-y} - e^{y} + c$

$2 \cos x + c_1 = y e^{-y} + e^{-y} + e^{y}$

$u = y \quad dv = e^{-y}$

$du = dy \quad v = -e^{-y}$

$-y e^{-y} + \int e^{-y} dy$

$-y e^{-y} - e^{-y}$

2.2

#35 $\frac{dy}{dx} = \sin x(\cos 2y - \cos^2 y)$

$\frac{dy}{\cos 2y - \cos^2 y} = \sin x \, dx$

$\frac{dy}{\cos^2 y - \sin^2 y - \cos^2 y} = \sin x \, dx$

$-\csc^2 y \, dy = \sin x \, dx$

$\cot y = -\cos x + c$

#41 $(e^{-y}+1)\sin x \, dx = (1+\cos x)dy, \quad y(0)=0$

$\frac{\sin x \, dx}{1+\cos x} = \frac{dy}{1+e^{-y}}$

$-\ln|1+\cos x| = \int \cancel{\frac{(1-e^{y})\, dy}{(1+e^{-y})(1-e^{y})}}$

$= \cancel{\frac{1-e^{y}}{1-e^{y}+e^{-y}-1}}$

$= \int \frac{e^{y}}{e^{y}+1}\, dy$

$= \ln(e^{y}+1) + \ln c$

$\frac{1}{1+\cos x} = c(e^{y}+1)$

$\frac{1}{1+\cos 0} = c(e^{0}+1)$

$\frac{1}{2} = 2c$

$\frac{1}{4} = c$

$\frac{1}{1+\cos x} = \frac{1}{4}(e^{y}+1)$

$4 = (e^{y}+1)(1+\cos x)$

2.2

#37 $x\sqrt{1-y^2}\,dx = dy$

$$x\,dx = (1-y^2)^{-1/2}\,dy$$

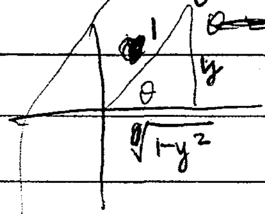
$$x\,dx = \frac{dy}{\sqrt{1-y^2}}$$

$dy = \cos\theta\, d\theta$

$y = \sin\theta$

~~θ = sin y~~ ~~y = sec θ~~ ~~θ = sin y~~

$$\frac{x^2}{2} = \int \frac{dy}{\sqrt{1-y^2}}$$




$$= \int \sec\theta \cos\theta \, d\theta$$

$$= \int d\theta$$

$$= \theta + c$$

$\cos\theta = \sqrt{1-y^2}$

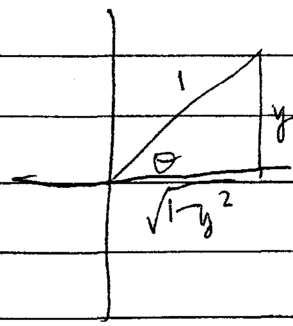
$\sec\theta = \frac{1}{\sqrt{1-y^2}}$

$$\frac{x^2}{2} = \arcsin y + c$$

$\theta = \arcsin y$

$$\frac{x^2}{2} + C = \arcsin y$$

$$y = \sin^{-1}\left(\frac{x^2}{2} + C\right)$$




#51 $\frac{dy}{dx} = x\sqrt{1-y^2}$

Let $y = 1$

This cannot be found from #37 with any value of c.

$y = \sin\theta$

$\cos\theta = \sqrt{1-y^2}$

$dy = \cos\theta\, d\theta$

22 # 45 $\frac{dx}{dy} = 4(x^2+1)$, $x\left(\frac{\pi}{4}\right) = 1$

$$\frac{dx}{x^2+1} = 4\,dy$$

$$\tan^{-1} x = 4y + C$$

$$\tan^{-1} 1 = 4\frac{\pi}{4} + C$$

$$\frac{\pi}{4} = \pi + C$$

$$-\frac{3\pi}{4} = C$$

$$\tan^{-1} x = 4y - \frac{3\pi}{4} \longrightarrow \boxed{x = \tan\left(4y - \frac{3\pi}{4}\right)}$$

OR $y = \frac{1}{4}\tan^{-1} x + \frac{3\pi}{16}$

2.2

#57 $\frac{dy}{dx} = (x+y+1)^2$

Let $u = ax + by + c$

$u = x + y + 1$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = u^2$

$du - dx = u^2 dx$

$du = u^2 dx + dx$

$\frac{du}{u^2+1} = dx$

$\tan^{-1} u = x + c$

$\tan x + c = u$

$\tan x + c = x + y + 1$

$y = -x - 1 + \tan(x+c)$

#59 $\frac{dy}{dx} = \tan^2(x+y)$

Let $u = x + y$

$\frac{dy}{dx} = \tan^2 u$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\frac{du}{dx} - 1 = \frac{dy}{dx}$

$\frac{du}{dx} - 1 = \tan^2 u$

$du - dx = \cancel{(\tan^2 u)}\, dx$

$du = (1 + \tan^2 u) dx$

$\frac{du}{1 + \tan^2 u} = dx$

$\frac{du}{\sec^2 u} = dx$

$\cos^2 u \, du = dx$

$\left(\frac{1 + \cos 2u}{2}\right) du = dx$

$\frac{u}{2} + \frac{\sin 2u}{4} = x + c$

$\frac{x+y}{2} + \frac{\sin(2x+2y)}{4} = x + c$

$2x + 2y + \sin(2x+2y) = 4x + c_1$

$2x + c_1 = 2y + \sin(2x+2y)$