EXERCISES 2.3 ZILL 5th

2.3. #1 $x^3 + 2xy^2 - y^4/x$

$$f(tx, ty) = t^3x^3 + 2txt^2y^2 - \frac{t^4y^4}{tx}$$

$$= t^3\left(x^3 + 2xy^2 - \frac{y^4}{x}\right)$$

$$= t^3 f(x, y)$$

homog. of degree 3

2.3 #3 $f(x, y) = \dfrac{x^3y - x^2y^2}{(x + 8y)^2}$

$$f(tx, ty) = \frac{t^3x^3yt - x^2t^2y^2t^2}{(xt + 8yt)^2}$$

$$= \frac{t^4(x^3y - x^2y^2)}{t^2(x + 8y)^2}$$

$$= t^2 f(x, y)$$

homogeneous of degree 2

2.3 #5 $\cos \dfrac{x^2}{x+y} = f(x, y)$

$$f(tx, ty) = \cos \frac{t^2x^2}{tx + ty}$$

$$= \cos \frac{tx^2}{x+y} \neq t^n \cos\left(\frac{x^2}{x+y}\right)$$

NOT Homogeneous.

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2.3 #7 $\ln x^2 - 2\ln y = f(x, y)$

$$f(tx, ty) = \ln t^2x^2 - 2\ln ty$$
$$= \ln t^2x^2 - \ln t^2y^2$$
$$= \ln \frac{t^2x^2}{t^2y^2}$$
$$= \ln \frac{x^2}{y^2}$$
$$= \ln x^2 - \ln y^2$$
$$= \ln x^2 - 2\ln y$$
$$= t^0 f(x, y)$$

Homogeneous of degree 0.

2.3. #11 $(x-y)dx + x\,dy = 0$

It is Homogeneous of degree 1

let $y = ux$ since $N(x,y) = x$ is simpler than $M(x,y) = x - y$

$$dy = u\,dx + x\,du$$

$$(x - ux)dx + x(u\,dx + x\,du) = 0$$

$$x(1-u)dx + xu\,dx + x^2 du = 0$$

$$[x(1-u) + xu]dx = -x^2 du$$

$$x(1 - u + u)dx = -x^2 du$$

$$x\,dx = -x^2 du$$

$$\frac{1}{x}dx = -du$$

$$\ln|x| = -u + C$$

$$\ln|x| + u = C$$

$$\ln|x| + \frac{y}{x} = C$$

$$x\ln|x| + y = Cx$$

2.3 #13 $x\,dx + (y-2x)\,dy = 0$

Yes Homog of degree 1

let $x = vy$

$dx = v\,dy + y\,dv$

$vy(v\,dy + y\,dv) + (y - 2vy)\,dy = 0$

$v^2 y\,dy + vy^2\,dv + y(1-2v)\,dy = 0$

$[v^2 y + y(1-2v)]\,dy + vy^2\,dv = 0$

$y(v^2 + 1 - 2v)\,dy + vy^2\,dv = 0$

$\frac{dy}{y} + \frac{v}{v^2 - 2v + 1}\,dv = 0$

$\ln|y| + \int\left(\frac{1}{v-1} + \frac{1}{(v-1)^2}\right)dv = C$

$\ln|y| + \ln|v-1| - (v-1)^{-1} = C$

$\ln|y| + \ln\left|\frac{x}{y} - 1\right| - \left(\frac{x}{y} - 1\right)^{-1} = C$

$\ln\left|y\left(\frac{x-y}{y}\right)\right| - \left(\frac{x-y}{y}\right)^{-1} = C$

$\ln|x-y| + \frac{y}{y-x} = C$

$\ln|x-y| - \frac{y}{x-y} = C$

$(x-y)\ln|x-y| - y = C(x-y)$

$\frac{v}{(v-1)^2} = \frac{A}{v-1} + \frac{B}{(v-1)^2}$

$v = A(v-1) + B$

$v = 1 \Rightarrow 1 = B$

$v = 2 \Rightarrow 2 = A + B$

$A = 1$

1

me

2.3#23 $\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$

$xy\,dy = (y^2 + x^2)dx$

$(y^2+x^2)dx - xy\,dy = 0$

Both Homogeneous of degree 2

Let $y = vx$

$dy = v\,dx + x\,dv$

$(v^2x^2 + x^2)dx - vx^2(v\,dx + x\,dv) = 0$

$x^2(v^2+1)dx - v^2x^2dx - vx^3dv = 0$

$x^2(v^2 - v^2 + 1)dx - vx^3dv = 0$

$x^2dx - vx^3dv = 0$

$\frac{dx}{x} - v\,dv = 0$

$\ln|x| - \frac{v^2}{2} = c_1$

$2\ln|x| - v^2 = 2c_1$

$\ln x^2 - v^2 = c$

$\ln x^2 - \frac{y^2}{x^2} = c$

2.3#31 $xy^2\frac{dy}{dx} = y^3 - x^3$, $y(1)=2$

$$xy^2\,dy + (x^3 - y^3)\,dx = 0$$

$$y = ux$$

$$dy = u\,dx + x\,du$$

$$x^3u^2\,dy + (x^3 - u^3x^3)\,dx = 0$$

$$x^3u^2(u\,dx + x\,du) + x^3(1-u^3)\,dx = 0$$

$$x^3u^3\,dx + x^4u^2\,du + x^3(1-u^3)\,dx = 0$$

$$x^4u^2\,du + x^3\,dx = 0$$

$$u^2\,du + \frac{dx}{x} = 0$$

$$u^2\,du = -\frac{dx}{x}$$

$$\frac{u^3}{3} = -\ln|x| + c$$

$$\frac{y^3}{3x^3} = -\ln|x| + c$$

$$y^3 = -3x^3\ln|x| + cx^3$$

$$2^3 = -3\ln|1| + c$$

$$8 = c$$

$$y^3 = -3x^3\ln|x| + 8x^3$$

2.3 #35 $(x+ye^{y/x})dx - xe^{y/x}dy = 0 \quad y(1)=0$

Homogeneous of degree 1

let $y = ux$

$dy = u\,dx + x\,du$

$(x+uxe^{u})dx - xe^{u}(u\,dx + x\,du) = 0$

$x(1+ue^{u})dx - uxe^{u}dx - x^{2}e^{u}du = 0$

$x\,dx - x^{2}e^{u}du = 0$

$\frac{dx}{x} = e^{u}du$

$\ln|x| = e^{u} + C$

$\ln|x| = e^{y/x} + C$

$\ln|1| = e^{0} + C$

$-1 = C$

$\ln|x| = e^{y/x} - 1$