EXERCISES 2.4 ZILL 5th

2.4#1 $(2x-1)dx + (3y+7)dy = 0$

$\frac{\partial M}{\partial y} = 0 \quad \frac{\partial N}{\partial x} = 0$ (EXACT)

$\frac{\partial f}{\partial x} = 2x-1$

$f = x^2 - x + g(y)$

$\frac{\partial f}{\partial y} = g'(y) = 3y+7$

$g(y) = \frac{3y^2}{2} + 7y$

$$\boxed{x^2 - x + \frac{3y^2}{2} + 7y = c}$$

2.4#3 $(5x+4y)dx + (4x-8y^3)dy = 0$

$\frac{\partial M}{\partial y} = 4 \quad \frac{\partial N}{\partial x} = 4$ (EXACT)

$\frac{\partial f}{\partial x} = 5x+4y$

$f = \frac{5x^2}{2} + 4xy + g(y)$

$\frac{\partial f}{\partial y} = 4x + g'(y) = 4x - 8y^3$

$g'(y) = -8y^3$

$g(y) = -2y^4$

$$\boxed{\frac{5x^2}{2} + 4xy - 2y^4 = c}$$

2.4#5 $(2y^2x-3)dx + (2yx^2+4)dy = 0$

$\frac{\partial M}{\partial y} = 4yx \qquad \frac{\partial N}{\partial x} = 4yx$ EXACT

$z = f(x,y) = c$

$\frac{\partial f}{\partial x} = 2y^2x - 3$

$f = y^2x^2 - 3x + g(y)$

$\frac{\partial f}{\partial y} = 2yx^2 + g'(y) = 2yx^2 + 4$

$g'(y) = 4$

$g(y) = 4y$

$$\boxed{y^2x^2 - 3x + 4y = c}$$

2.4#7 $(x+y)(x-y)dx + x(x-2y)dy = 0$

$(x^2-y^2)dx + (x^2-2xy)dy = 0$

$\frac{\partial M}{\partial y} = -2y \qquad \frac{\partial N}{\partial x} = 2x-2y$ NOT EXACT

2.4#9 $(y^3 - y^2\sin x - x)dx + (3xy^2 + 2y\cos x)dy = 0$

$\frac{\partial M}{\partial y} = 3y^2 - 2y\sin x \qquad \frac{\partial N}{\partial x} = 3y^2 - 2y\sin x$ EXACT

$\frac{\partial f}{\partial x} = y^3 - y^2\sin x - x$

$f = xy^3 + y^2\cos x - \frac{x^2}{2} + g(y)$

$\frac{\partial f}{\partial y} = 3xy^2 + 2y\cos x + g'(y) = 3xy^2 + 2y\cos x$

$g'(y) = 0$

$g(y) = c_1$

$$\boxed{xy^3 + y^2\cos x - \frac{x^2}{2} = c}$$

2.4 #11 $(y\ln y - e^{-xy})dx + \left(\frac{1}{y} + x\ln y\right)dy = 0$

$\frac{\partial M}{\partial y} = \ln y + 1 + xe^{-xy}$, $\frac{\partial N}{\partial x} = \ln y$ NOT EX.

2.4 #13 $x\frac{dy}{dx} = 2xe^x - y + 6x^2$

$x\,dy = (2xe^x - y + 6x^2)dx$

$(2xe^x - y + 6x^2)dx - x\,dy = 0$

$\frac{\partial M}{\partial y} = -1$ $\frac{\partial N}{\partial x} = -1$ EXACT

$\frac{\partial f}{\partial x} = 2xe^x - y + 6x^2$

$f = 2xe^x - 2e^x - xy + 2x^3 + g(y)$

$\frac{\partial f}{\partial y} = -x + g'(y) = -x$

$g'(y) = 0$

$g(y) = c_1$

$$\boxed{2xe^x - 2e^x - xy + 2x^3 = C}$$

2.4 #15

$\left(1 - \frac{3}{x} + y\right)dx + \left(1 - \frac{3}{y} + x\right)dy = 0$

$\frac{\partial M}{\partial y} = 1$, $\frac{\partial N}{\partial x} = 1$ EXACT

$\frac{\partial f}{\partial x} = 1 - \frac{3}{x} + y$

$f = x - 3\ln|x| + xy + g(y)$

$\frac{\partial f}{\partial y} = x + g'(y) = 1 - \frac{3}{y} + x$

$g'(y) = 1 - \frac{3}{y}$

$g(y) = y - 3\ln|y|$

$x - 3\ln|x| + xy + y - 3\ln|y| = C$

$$\boxed{x + y + xy - 3\ln|xy| = C}$$

2.4 #19 $\left(x^2y^3 - \frac{1}{1+9x^2}\right)dx + x^3y^2\,dy = 0$

$\frac{\partial M}{\partial y} = 3x^2y^2 \qquad \frac{\partial N}{\partial x} = 3x^2y^2$ EXACT

$\frac{\partial f}{\partial x} = x^2y^3 - \frac{1}{1+9x^2}$

$f = \frac{x^3y^3}{3} - \frac{1}{3}\tan^{-1}3x + g(y)$

$\frac{\partial f}{\partial y} = x^3y^2 + g'(y) = x^3y^2$

$g'(y) = 0$

$g(y) = c_1$

$\frac{x^3y^3}{3} - \frac{1}{3}\tan^{-1}3x = c_2$

$x^3y^3 - \tan^{-1}3x = C$

2.4 #25 $(x+y)^2dx + (2xy + x^2 - 1)dy = 0 \qquad y(1) = 1$

$\frac{\partial M}{\partial y} = 2(x+y) \qquad \frac{\partial N}{\partial x} = 2x + 2y$ EXACT

$\frac{\partial f}{\partial x} = (x+y)^2$

$f = \frac{(x+y)^3}{3} + g(y)$

$\frac{\partial f}{\partial y} = (x+y)^2 + g'(y) = x^2 + 2xy - 1 + y^2 - y^2$

$g'(y) = -1 - y^2$

$g(y) = -y - \frac{y^3}{3}$

$\frac{(x+y)^3}{3} - y - \frac{y^3}{3} = c$

$\frac{(1+1)^3}{3} - 1 - \frac{1}{3} = c$

$\frac{8}{3} - \frac{3}{3} - \frac{1}{3} = c$

$\frac{4}{3} = c$

$\frac{(x+y)^3}{3} - y - \frac{y^3}{3} = \frac{4}{3}$

$\boxed{(x+y)^3 - 3y - y^3 = -4}$

2.4 #29 $(y^2\cos x - 3x^2y - 2x)dx + (2y\sin x - x^3 + \ln y)dy = 0$

$y(0) = e$

$\frac{\partial M}{\partial y} = 2y\cos x - 3x^2$, $\frac{\partial N}{\partial x} = 2y\cos x - 3x^2$ EXACT

$\frac{\partial f}{\partial x} = y^2\cos x - 3x^2y - 2x$

$f = y^2\sin x - x^3y - x^2 + g(y)$

$\frac{\partial f}{\partial y} = 2y\sin x - x^3 + g'(y) = 2y\sin x - x^3 + \ln y$

$g'(y) = \ln y$

$g(y) = \int \ln y\, dy$

$= y\ln y - y$

$y^2\sin x - x^3y - x^2 + y\ln y - y = c$

$e^2\sin 0 - 0 - 0 + e\ln e - e = c$

$0 = c$

$$\boxed{y^2\sin x - x^3y - x^2 + y\ln y - y = 0}$$

2-4 #35 $M(x,y)\,dx + \left(xe^{xy} + 2xy + \frac{1}{x}\right)dy = 0$

Find $M(x,y)$ so that this eq. is exact

$$\frac{\partial N}{\partial x} = e^{xy} + yxe^{xy} + 2y - \frac{1}{x^2} = \frac{\partial M}{\partial y}$$

$$M = \int e^{xy} + xye^{xy} + 2y - x^{-2}\,dy$$

$$= \frac{1}{x}e^{xy} + ye^{xy} - \frac{1}{x}e^{xy} + y^2 - yx^{-2}$$

$$= ye^{xy} + y^2 - \frac{y}{x^2}$$

$\int xye^{xy}\,dy$

$u = y \qquad dv = xe^{xy}\,dy$

$du = dy \qquad v = e^{xy}$

$ye^{xy} - \int e^{xy}\,dy$

$ye^{xy} - \frac{1}{x}e^{xy}$