2.7
#3 $-y\,dx + (1+ye^x)\,dy = 0$

$u = ye^x$

$y = \frac{u}{e^x}$

$dy = \frac{e^x du - ue^x dx}{e^{2x}}$

$dy = \frac{du - u\,dx}{e^x}$

$\frac{u}{e^x}dx + (1+u)\left(\frac{du - u\,dx}{e^x}\right) = 0$

$u\,dx + (1+u)du - u(1+u)dx = 0$

$(u - u(1+u))dx = -(1+u)du$

$-u^2 dx = -(1+u)du$

$dx = \frac{1+u}{u^2}du$

$\int dx = \int (u^{-2} + u^{-1})\,du$

$x + C = -u^{-1} + \ln|u|$

$x + C = \frac{-1}{ye^x} + \ln|ye^x|$

$x + C = \frac{1}{ye^x} + \ln|y| + \ln e^x$ (→ x)

$cy = -e^{-x} + y\ln|y|$

$cy + e^{-x} = y\ln|y|$

2.7 #13 $y' + 1 = e^{-(x+y)} \sin x$

Let $u = e^{-(x+y)} = e^{-x} \cdot e^{-y}$

$du = -e^{-x} e^{-y} dx - e^{-x} e^{-y} dy$

$\frac{du}{dx} = -u - u\frac{dy}{dx}$

$\frac{du}{dx} + u \cdot = -u\frac{dy}{dx}$

$-\frac{1}{u}\frac{du}{dx} - 1 = \frac{dy}{dx}$

$-\frac{1}{u}\frac{du}{dx} - 1 + 1 = u \sin x$

$-\frac{1}{u}\frac{du}{dx} = u \sin x$

$-\frac{1}{u^2} du = \sin x \, dx$

$u^{-1} = -\cos x + C$

$e^{x+y} = -\cos x + C$

$$y\frac{dx}{dy} + 2x\ln x = xe^y$$

Let $u = \ln x \longrightarrow e^u = x$

$$\frac{dx}{dy} + \frac{2x\ln x}{y} = \frac{xe^y}{y} \qquad \frac{dx}{dy} = e^u \cdot \frac{du}{dy}$$

$$y e^u \frac{du}{dy} + \frac{2e^u u}{y} = \frac{e^u e^y}{y}$$

$$y e^u \frac{du}{dy} + 2ue^u = e^u e^y$$

$$y\frac{du}{dy} + 2u = e^y$$

$$\frac{du}{dy} + 2\frac{u}{y} = e^y y^{-1}$$

Linear in u

$$\mu(y) = e^{\int \frac{2}{y} dy} = e^{2\ln y} = y^2$$

$$y^2\frac{du}{dy} + 2uy = ye^y$$

$$\frac{d}{dy}[y^2 u] = ye^y$$

$$y^2 u = \int ye^y dy$$

$$y^2 u = ye^y - e^y + c$$

$$y^2 \ln x = ye^y - e^y + c$$

$$y^2 = \frac{ye^y}{\ln x} - \frac{e^y}{\ln x} + \frac{c}{\ln x}$$

27. # 17 $y'' + (y')^2 + 1 = 0$

Let $y' = u \Rightarrow y'' = u'$

$u' + u^2 + 1 = 0$

$\frac{du}{dx} = -(u^2+1)$

$du = (u^2+1)\,dx$

$\frac{du}{u^2+1} = -dx$

$\tan^{-1} u = -x + C_1$

$u = \tan(C_1 - x)$

$y' = \tan(C_1 - x)$

$dy = \tan(C_1 - x)\,dx$

$y = \ln|\cos(C_1 - x)| + C_2$