Solutions 3.1 & 3.2 Monbrod
Zill 5th Ed.

3.1 #1 $y = c_1 x$

$$\frac{dy}{dx} = c_1 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$y\,dy = -x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c_2$$

$y^2 + x^2 = c_2$ ORTHOGONAL TO $y = c_1 x$

#3 $y = c_1 x^2$

$$\frac{dy}{dx} = 2c_1 x \Rightarrow \frac{dy}{dx} = 2\frac{y}{x^2}\cdot x$$

$$\frac{dy}{dx} = -\frac{x}{2y}$$

$$2y\,dy = -x\,dx$$

$$y^2 = -\frac{x^2}{2} + c_2$$

$$2y^2 + x^2 = c_2$$

$x\,b =$

\# 5 $c_1 x^2 + y^2 = 1 \Rightarrow c_1 = \frac{1-y^2}{x^2}$

$$2c_1 x + 2y\frac{dy}{dx} = 0$$

$$c_1 x + y\frac{dy}{dx} = 0$$

$$\frac{1-y^2}{x^2}\cdot x + y\frac{dy}{dx} = 0$$

$$\frac{1-y^2}{x} = -y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y^2-1}{xy}$$

SO $\frac{dy}{dx} = -\frac{xy}{y^2-1}$ will yield $\perp$ Family

$$\frac{(y^2-1)}{y}dy = -x\,dx$$

$$\int\left(y - \frac{1}{y}\right)dy = \int -x\,dx$$

$$\frac{y^2}{2} - \ln|y| = -\frac{x^2}{2} + c_2$$

$$y^2 - 2\ln|y| = -x^2 + c_2$$

$$c_2 + y^2 + x^2 = 2\ln|y|$$

3.1 #7 $y = c_1 e^{-x} \Rightarrow c_1 = e^x y$

$$\frac{dy}{dx} = -c_1 e^{-x}$$

$$= -e^x y e^{-x}$$

$$\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{1}{y}$$

$$y\,dy = dx$$

$$\frac{y^2}{2} = x + c_2$$

$$y^2 - 2x = c_2$$

1 # 21 $y = \frac{1}{\ln c_1 x} = (\ln c_1 x)^{-1}$ $\quad \ln c_1 x = y^{-1}$

$$e^{y^{-1}} = c_1 x$$

$$c_1 = \frac{e^{\frac{1}{y}}}{x}$$

$$\frac{dy}{dx} = -(\ln c_1 x)^{-2}\left(\frac{c_1}{c_1 x}\right)$$

$$= \frac{-1}{x(\ln c_1 x)^2} = \frac{-1}{x(\ln \frac{e^{1/y}}{x} x)^2} = \frac{-1}{x(\frac{1}{y})^2} = -\frac{y^2}{x}$$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$y^2 dy = x\,dx$$

$$\frac{y^3}{3} = \frac{x^2}{2} + c_2$$

$$2y^3 - 3x^2 = c_2$$

3.2 #1 $P = P_0 e^{kt}$ FROM $\frac{dP}{dt} = kP$

①

$2P_0 = P_0 e^{5k}$

$2 = e^{5k}$

$\ln 2 = 5k$

$\frac{\ln 2}{5} = k$

$P = P_0 e^{(\frac{1}{5}\ln 2)t}$

$\frac{dP}{P} = k\,dt$

$\ln P = kt + C$

$t=0 \Rightarrow \ln P_0 = C$

$e^C = P_0$

$e^{kt+C} = P$

$e^C e^{kt} = P$

$P = P_0 e^{kt}$

② $3P_0 = P_0 e^{\frac{1}{5}\ln 2 \cdot t}$

$\ln 3 = (\frac{1}{5}\ln 2)t$

$\frac{5\ln 3}{\ln 2} = t$

$7.9 = t$ to triple

$4P_0 = P_0 e^{(\frac{1}{5}\ln 2)t}$

$\frac{5\ln 4}{\ln 2} = t$

$10 = t$ to quadruple

2 #3 $P = P_0 e^{kt}$ $P_0 = 500$

$1.15(500) = 500e^{10k}$

$1.15 = e^{10k}$

$\frac{\ln 1.15}{10} = k$

$P(30) = 500\, e^{\frac{1}{10}\ln 1.15(30)}$

$= 760.4$

3.2 #5 $\frac{dN}{dt} = kN$ $N(3.3\text{hrs}) = \frac{1}{2}N_0$, $N_0 = 1$

$= \frac{1}{2}$

$\downarrow$

$\frac{dN}{N} = k\,dt$

$\ln N = kt + C$

$e^{kt+C} = N \Rightarrow N = e^C \cdot e^{kt}$

$N(0) = 1 = e^C$

$N = e^{kt}$

$\frac{1}{2} = N(3.3) = e^{3.3k}$

$\ln .5 = 3.3k$

$\frac{\ln .5}{3.3} = k$

$.1(1) = N(t) = 1e^{kt}$

$\ln(.1) = kt$

$\ln .1 = \frac{\ln .5}{3.3} t$

$t = \frac{\ln .1}{\frac{\ln .5}{3.3}}$

$t = 10.96$ hrs

3.2 #13

Time	T inside	T outside
0	70°	10°
1/2	50°	10°
1	?	10°
?	15°	10°

$$T = T_m + (T_0 - T_m)e^{kt} = 10 + 60e^{kt}$$

$$50^\circ = 10 + 60e^{k/2}$$

$$40^\circ = 60e^{k/2}$$

$$\frac{2}{3} = e^{k/2}$$

$$\ln \tfrac{2}{3} = k/2$$

$$2\ln\tfrac{2}{3} = k$$

$$T = 10 + 60e^{t\ln 4/9}$$

$$t = 1 \Rightarrow T = 10 + 60e^{\ln 4/9} = 10 + 60\cdot\frac{4}{9}$$

$$\boxed{= 36.67^\circ}$$

$$15^\circ = 10 + 60e^{t\ln 4/9}$$

$$\frac{5^\circ}{60^\circ} = e^{t\ln 4/9}$$

$$\ln\frac{1}{12} = t\ln 4/9$$

$$\frac{\ln\frac{1}{12}}{\ln 4/9} = t$$

$$\boxed{3.06 = t}$$

3.2 # 21

$$\frac{dA}{dt} = R_{in} - R_{out}$$

$$R_{in} = \left(\frac{1 \text{ gram}}{1 \text{ liter}}\right) \cdot \left(\frac{4 \text{ liters}}{\text{min}}\right) = \frac{4 \text{ grams}}{\text{min}}$$

$$R_{out} = \left(\frac{A \text{ gram}}{200 \text{ L}}\right)\left(\frac{4 \text{ liters}}{\text{min}}\right)$$

$$\frac{dA}{dt} = 4 - 0.02A$$

$$\frac{dA}{dt} + 0.02A = 4$$

$$\mu(t) = e^{\int 0.02\,dt}$$

$$= e^{0.02t}$$

$$e^{0.02t}\frac{dA}{dt} + .002e^{0.02t}A = 4e^{0.02t}$$

$$\frac{d}{dt}\left[Ae^{0.02t}\right] = 4e^{0.02t}$$

$$Ae^{0.02t} = \int 4e^{0.02t}\,dt$$

$$Ae^{0.02t} = \frac{4}{0.02}e^{0.02t} + C$$

$$A = 200 + Ce^{-0.02t}$$

$$A(0) = 30 = 200 + C$$

$$C = -170$$

$$A = 200 - 170e^{-0.02t}$$

1. #25

$$\frac{dA}{dt} = R_{in} - R_{out}$$

$$R_{in} = \frac{\frac{1}{2}\,Lb}{gal} \cdot \frac{6\,gal}{min} = \frac{3\,Lb}{1\,min}$$

$$R_{out} = \frac{A\,Lb/gal}{100+2t} \cdot \frac{4\,gal}{min}$$

$$\frac{dA}{dt} = 3 - \frac{4A}{100+2t}$$

$$\frac{dA}{dt} = 3 - \frac{2A}{50+t}$$

$$\frac{dA}{dt} + \frac{2}{50+t}A = 3 \qquad \mu(t) = e^{\int \frac{2}{50+t}dt} = e^{2\ln 50+t} = (50+t)^2$$

$$(50+t)^2\frac{dA}{dt} + \frac{2(50+t)^2}{50+t}A = 3(50+t)^2$$

$$\frac{d}{dt}\left[A(50+t)^2\right] = 3(50+t)^2$$

$$d\left[A(50+t)^2\right] = 3(50+t)^2\,dt$$

$$A(50+t)^2 = 3\int(50+t)^2\,dt$$

$$A(50+t)^2 = (50+t)^3 + C$$

$$A = 50+t + C(50+t)^{-2}$$

$$A(0) = 10 = 50 + \frac{C}{50^2}$$

$$-40 = \frac{C}{50^2}$$

$$C = -40 \cdot 2500$$

$$= -100{,}000$$

$$A = 50 + t - 100000(50+t)^{-2}$$

$$A(30) = 50 + 30 - 100000(80)^{-2}$$

$$A = 64.375$$

302 #31

$$\frac{dP}{dt} = \frac{dB}{dt} - \frac{dD}{dt}$$

$$\frac{dB}{dt} = k_1 P \qquad \frac{dD}{dt} = k_2 P$$

a) $$\frac{dP}{dt} = k_1 P - k_2 P$$

$$\frac{dP}{dt} = P(k_1 - k_2)$$

$$\frac{dP}{P} = k_1 - k_2)dt$$

$$\ln P = (k_1 - k_2)t + c$$

$$P = e^{(k_1 - k_2)t + c}$$

$$P(0) = P_0 \Rightarrow P_0 = e^{c}$$

$$P = P_0 e^{(k_1 - k_2)t}$$

5) $k_1 > k_2$ Population grows

$k_1 < k_2$ Population shrinks

$k_1 = k_2$ Population remains stagnant.