Math 205 Monbrod

Selected Solutions Zill 5th

4.4

4.4#1 $y'' + 3y' + 2y = 6$

$m^2 + 3m + 2 = 0$

$(m+2)(m+1) = 0$

$y_c = c_1 e^{-2x} + c_2 e^{-x}$

$y_p = A$, $y_p' = 0$, $y_p'' = 0$

$2A = 6 \Rightarrow A = 3$

$y_p = A = 3$

$y = c_1 e^{-2x} + c_2 e^{-x} + 3$

4.4#3 $y'' - 10y' + 25y = 30x + 3$

$m^2 - 10m + 25 = 0$

$(m-5)^2 = 0$

$y_c = c_1 e^{5x} + c_2 x e^{5x}$

$y_p = Ax + B$, $y_p' = A$, $y_p'' = 0$

$0 - 10A + 25(Ax + B) = 30x + 3$

$-10A + 25Ax + 25B = 30x + 3$

$25A = 30$, $-10A + 25B = 3$

$A = \frac{6}{5}$

$25B = 3 + 10A$

$= 3 + 12$

$25B = 15$

$B = \frac{15}{25} = \frac{3}{5}$

$y = y_c + y_p = c_1 e^{5x} + c_2 x e^{5x} + \frac{6}{5}x + \frac{3}{5}$

4.4#9 $y'' - y' = -3$

$m^2 - m = 0$

$m(m-1)$

$m = 0, m = 1$

$y_c = C_1 + C_2 e^x$

Let $y_p = Ax$

$y_p' = A$

$y_p'' = 0$

$0 - A = -3$

$A = 3$

$y = c_1 + C_2 e^x + 3x$

4.4# 11 $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$

$m^2 - m + \frac{1}{4} = 0$

$(m - \frac{1}{2})^2 = 0$

$m = \frac{1}{2}$

$y_c = C_1 e^{\frac{1}{2}x} + C_2 x e^{\frac{1}{2}x}$ $\quad y_p = A + Bx^2 e^{x/2}$

$y_p' = 2Bx e^{x/2} + \frac{B}{2}x^2 e^{x/2}$

$y_p'' = 2Be^{x/2} + Bx e^{x/2} + Bx e^{x/2} + \frac{B}{4}x^2 e^{x/2} = 2Be^{x/2} + 2Bx e^{x/2} + \frac{Bx^2}{4}e^{x/2}$

$2Be^{x/2} + 2Bx e^{x/2} + \frac{Bx^2}{4}e^{x/2} - 2Bx e^{x/2} - \frac{B}{2}x^2 e^{x/2} + \frac{A}{4} + \frac{B}{4}x^2 e^{x/2} = 3 + e^{x/2}$

$\frac{A}{4} = 3 \qquad 2B = 1$

$A = 12 \qquad B = \frac{1}{2}$

$y = C_1 e^{\frac{1}{2}x} + C_2 x e^{\frac{1}{2}x} + 12 + \frac{1}{2}x^2 e^{x/2}$

4.4 #15

$y'' + y = 2x\sin x \longrightarrow y_p = (Ax+B)\sin x + (Cx+D)\cos x$

$m^2 + 1 = 0$

$m = \pm i$

$y_c = c_1\cos x + c_2\sin x$ $\Big\}$ $y_p = (Ax^2+Bx)\sin x + (Cx^2+Dx)\cos x$

$y_p' = (2Ax+B)\sin x + (Ax^2+Bx)\cos x + (2Cx+D)\cos x - (Cx^2+Dx)\sin x$

$y_p' = (-Cx^2+(2A-D)x+B)\sin x + (Ax^2+(B+2C)x+D)\cos x$

$y_p'' = (-2Cx+2A-D)\sin x + (-Cx^2+(2A-D)x+B)\cos x + (2Ax+B+2C)\cos x$

$-(Ax^2+(B+2C)x+D)\sin x$

$= (-Ax^2-(B+4C)x+2A-2D)\sin x + (-Cx^2+(4A-D)x+2B+2C)\cos x$

$y_p'' + y = 2x\sin x$

$(-Ax^2-(B+4C)x+2A-2D)\sin x + (-Cx^2+(4A-D)x+2B+2C)\cos x + (Ax^2+Bx)\sin x$

$+(Cx^2+Dx)\cos x = 2x\sin x$

$-Ax^2-(B+4C)x+2A-2D+Ax^2+Bx = 2x$, $\quad -Cx^2+(4A-D)x+2B+2C+Cx^2+Dx = 0$

$-Bx-4Cx+Bx = 2x$

$-4C = 2$

$C = -\frac{1}{2}$

$2A-2D = 0$

$D = A$

$Dx+4Ax+2B+2C = 0$

$D+4A = 0, \quad 2B+2C = 0$

$D-4D = 0 \quad 2B = -2C$

$D = 0 \quad B = -C$

$\Rightarrow A = 0 \quad B = \frac{1}{2}$

$y = c_1\cos x + c_2\sin x + \frac{1}{2}x\sin x - \frac{1}{2}x^2\cos x$