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1-11

$$x^2 + y^2 = z^2$$

$$x^2 + y^2 = (x+y)^2 - 2xy = z^2$$

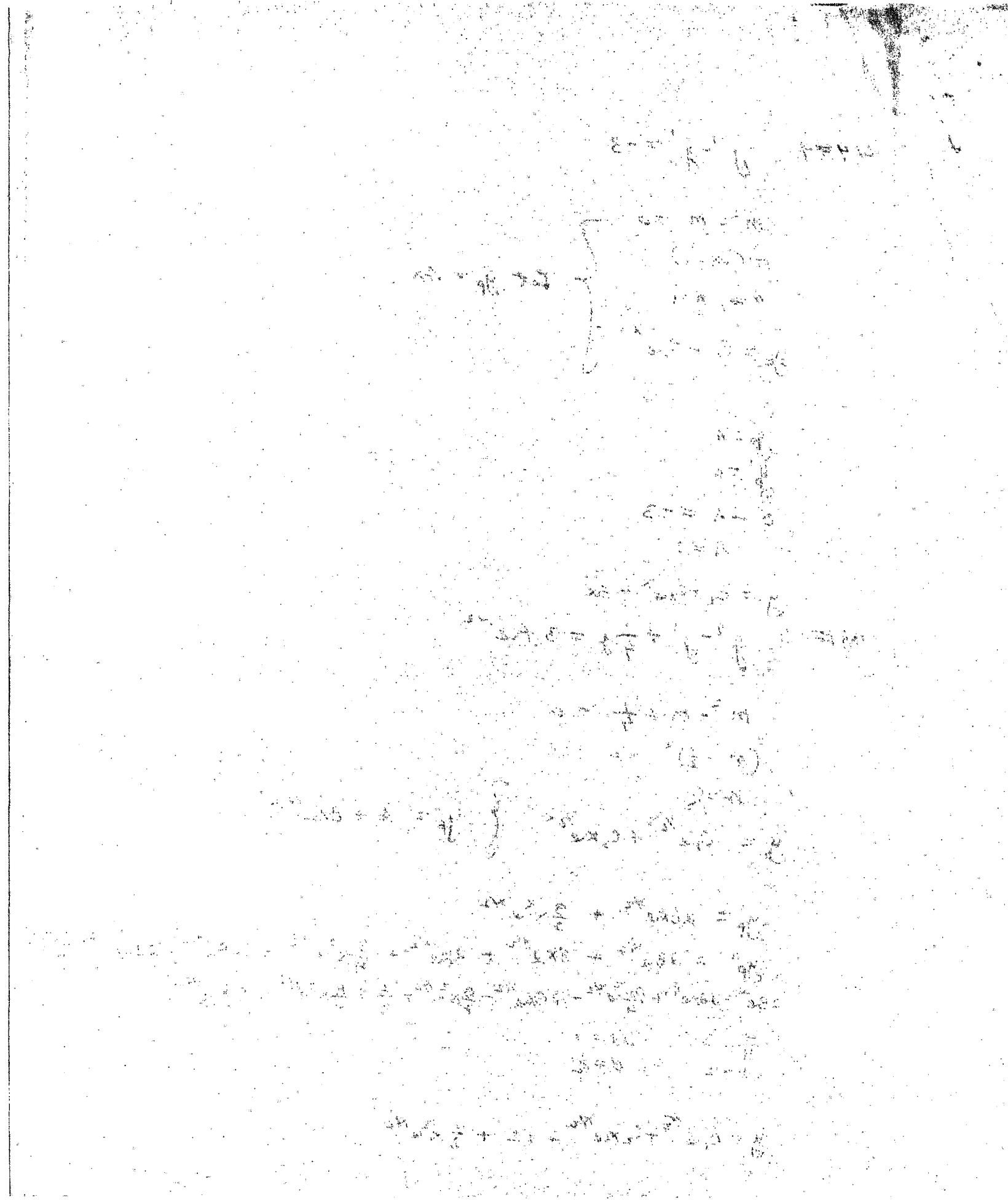
$$2xy = (x+y)^2 - z^2 = (x+y+z)(x+y-z)$$

$$2xy = (x+y+z)(x+y-z)$$

$$2xy = (x+y)^2 - z^2$$

$$2xy = (x+y+z)(x+y-z)$$

$$x^2 + y^2 = z^2$$



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Main body of handwritten text, appearing to be a list or series of notes. A circled symbol is visible in the middle of this section.

Section of handwritten text containing several lines of what appear to be mathematical or technical notations.

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4.4#21

$$y'''' - 6y'' = 3 - \cos x \longrightarrow y_p = A + B \cos x + C \sin x$$

$$m^3 - 6m^2 = 0$$

$$m^2(m-6) = 0$$

$$m=0, m=6$$

$$y_c = c_1 + c_2 x + c_3 e^{6x} \longrightarrow y_p = Ax^2 + B \cos x + C \sin x$$

$$y_p' = 2Ax - B \sin x + C \cos x$$

$$y_p'' = 2A - B \cos x - C \sin x$$

$$y_p''' = B \sin x - C \cos x$$

$$B \sin x - C \cos x - 6(2A - B \cos x - C \sin x) = 3 - \cos x$$

$$B \sin x - C \cos x - 12A + 6B \cos x + 6C \sin x = 3 - \cos x$$

$$(B+6C) \sin x + (6B-C) \cos x - 12A = 3 - \cos x$$

$$B+6C=0, \quad 6B-C=-1, \quad -12A=3$$

$$B=-6C, \quad 6(-6C)-C=-1, \quad A=-\frac{1}{4}$$

$$-37C=-1$$

$$C = \frac{1}{37} \Rightarrow B = -\frac{6}{37}$$

$$y = c_1 + c_2 x + c_3 e^{6x} - \frac{1}{4} x^2 - \frac{6}{37} \cos x + \frac{1}{37} \sin x$$

4.4 #23

$$y''' - 3y'' + 3y' - y = x - 4e^x$$

$$(m-1)^3 = 0$$

$$y_c = C_1 e^x + C_2 x e^x + C_3 x^2 e^x$$

$$y_p = Ax + B + 4C e^x$$

$$\rightarrow y_p = Ax + B + 4Cx^3 e^x$$

(Used a 4C by accident only C is needed)

$$y_p' = A + 12Cx^2 e^x + 4Cx^3 e^x$$

$$y_p'' = 24Cxe^x + 12Cx^2 e^x + 12Cx^2 e^x + 4Cx^3 e^x$$

$$= 24Cxe^x + 24Cx^2 e^x + 4Cx^3 e^x$$

$$y_p''' = 24Ce^x + 24Cxe^x + 48Cxe^x + 24Cx^2 e^x + 12Cx^2 e^x + 4Cx^3 e^x$$

$$= 24Ce^x + 72Cxe^x + 12Cx^2 e^x + 4Cx^3 e^x$$

$$y'' - 3y' + 3y' - y = x - 4e^x$$

$$24Ce^x + 72Cxe^x + 36Cx^2 e^x + 4Cx^3 e^x - 72Cxe^x - 72Cx^2 e^x - 12Cx^3 e^x + 3A + 36Cx^2 e^x + 12Cx^3 e^x - Ax - B - 4Cx^3 e^x = x - 4e^x$$

$$24C = -4$$

$$C = -\frac{1}{6}$$

$$-A = 1$$

$$A = -1$$

$$3A - B = 0$$

$$B = 3A$$

$$B = -3$$

$$y = C_1 e^x + C_2 x e^x + C_3 x^2 e^x - x - 3 - \frac{1}{6} x^3 e^x$$

$$H.4 \#27 \quad y'' + y = 8 \sin^2 x = 8 \left(\frac{1 - \cos 2x}{2} \right) = 4 - 4 \cos 2x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$y_c = C_1 \cos x + C_2 \sin x$$

$$y_p = A + B \cos 2x + C \sin 2x$$

$$y_p' = -2B \sin 2x + 2C \cos 2x$$

$$y_p'' = -4B \cos 2x - 4C \sin 2x$$

$$-4B \cos 2x - 4C \sin 2x + A + B \cos 2x + C \sin 2x = 4 - 4 \cos 2x$$

$$A - 3B \cos 2x - 3C \sin 2x = 4 - 4 \cos 2x$$

$$A = 4, \quad -3B = -4, \quad -3C = 0$$

$$B = \frac{4}{3}, \quad C = 0$$

$$y = C_1 \cos x + C_2 \sin x + \underbrace{4 + \frac{4}{3} \cos 2x}_{y_p}$$

$$4.4 \# 29. \quad y'' + 4y = -2, \quad y\left(\frac{\pi}{8}\right) = \frac{1}{2}, \quad y'\left(\frac{\pi}{8}\right) = 2$$

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$y_h = c_1 \cos 2x + c_2 \sin 2x$$

$$y_p = A$$

$$4A = -2$$

$$A = -\frac{1}{2}$$

$$y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{2}$$

$$y\left(\frac{\pi}{8}\right) = \frac{1}{2} = c_1 \cos \frac{\pi}{4} + c_2 \sin \frac{\pi}{4} - \frac{1}{2}$$

$$\textcircled{1} \quad 1 = \frac{c_1 \sqrt{2}}{2} + \frac{c_2 \sqrt{2}}{2}$$

$$y' = -2c_1 \sin 2x + 2c_2 \cos 2x$$

$$y'\left(\frac{\pi}{8}\right) = 2 = -2c_1 \sin \frac{\pi}{4} + 2c_2 \cos \frac{\pi}{4}$$

$$\textcircled{2} \quad 1 = -\frac{c_1 \sqrt{2}}{2} + \frac{c_2 \sqrt{2}}{2}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow 2 = \frac{2c_2 \sqrt{2}}{2}$$

$$\frac{2}{\sqrt{2}} = c_2$$

$$\frac{2\sqrt{2}}{2} = c_2$$

$$\sqrt{2} = c_2$$

$$\textcircled{1} \Rightarrow 2 = c_1 \sqrt{2} + 2$$

$$c_1 = 0$$

$$y = \sqrt{2} \sin 2x - \frac{1}{2}$$

$$4.4 \# 31. \quad 5y'' + y' = -6x$$

$$y(0) = 9, \quad y'(0) = -10$$

$$5m^2 + m = 0$$

$$m(5m + 1) = 0$$

$$m = 0, \quad m = -\frac{1}{5}$$

$$y_c = c_1 + c_2 e^{-\frac{x}{5}}$$

$$y_p = 4x + B$$

$$y_p = Ax^2 + Bx$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$5y'' + y' = -6x$$

$$10A + 2Ax + B = -6x$$

$$10A + B = 0, \quad 2A = -6$$

$$A = -3$$

 \Rightarrow

$$-30 + B = 0$$

$$B = 30$$

$$y = c_1 + c_2 e^{-x/5} - 3x^2 + 30x \quad \Rightarrow \quad y' = -\frac{c_2}{5} e^{-x/5} - 6x + 30$$

$$y(0) = c_1 + c_2 = 0, \quad y'(0) = -\frac{c_2}{5} + 30 = -10$$

$$c_2 = -c_1$$

$$-c_2 = -50 - 150$$

$$c_2 = 200$$

$$y = 200e^{-x/5} - 3x^2 + 30x - 200$$

$$4.4 \#37 \quad y'' + y = \cos x - \sin 2x, \quad y\left(\frac{\pi}{2}\right) = 0, \quad y'\left(\frac{\pi}{2}\right) = 0$$

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$y_c = C_1 \cos x + C_2 \sin x$$

$$y_p = A \cos x + B \sin x + C \sin 2x + D \cos 2x$$

$$\text{use } y_p = Ax \cos x + Bx \sin x + C \sin 2x + D \cos 2x$$

$$y_p' = A \cos x - Ax \sin x + B \sin x + Bx \cos x + 2C \cos 2x - 2D \sin 2x$$

$$y_p'' = -A \sin x - A \sin x - Ax \cos x + B \cos x - Bx \sin x + B \cos x - 4C \sin 2x - 4D \cos 2x$$

$$y_p'' + y_p \text{ becomes } -2A \sin x - Bx \sin x - Ax \cos x + 2B \cos x - 4C \sin 2x - 4D \cos 2x$$

$$+ Ax \cos x + Bx \sin x + C \sin 2x + D \cos 2x = \cos x - \sin 2x$$

$$-2A \sin x + 2B \cos x - 3C \sin 2x - 3D \cos 2x = \cos x - \sin 2x$$

$$-2A = 0, \quad 2B = 1, \quad -3C = -1, \quad -3D = 0$$

$$A = 0, \quad B = \frac{1}{2}, \quad C = \frac{1}{3}, \quad D = 0$$

$$y = C_1 \cos x + C_2 \sin x + \frac{1}{2} x \sin x + \frac{1}{3} \sin 2x$$

$$y\left(\frac{\pi}{2}\right) = C_2 + \frac{\pi}{4} = 0$$

$$C_2 = -\frac{\pi}{4}$$

$$y' = -C_1 \sin x + C_2 \cos x + \frac{1}{2} \sin x + \frac{1}{2} x \cos x + \frac{2}{3} \cos 2x$$

$$y'\left(\frac{\pi}{2}\right) = -C_1 + \frac{1}{2} - \frac{2}{3} = 0$$

$$C_1 = \frac{1}{2} - \frac{2}{3} = -\frac{1}{6}$$

$$y = -\frac{1}{6} \cos x - \frac{\pi}{4} \sin x + \frac{1}{2} x \sin x + \frac{1}{3} \sin 2x$$

$$4.4 \text{ \#41. } y'' + y = x^2 + 1, \quad y(0) = 5, \quad y(\pi) = 0$$

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$y = c_1 \cos x + c_2 \sin x$$

$$\Rightarrow y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$2A + Ax^2 + Bx + C = x^2 + 1$$

$$2A + C = 1, \quad A = 1, \quad B = 0$$

$$C = -1$$

$$y_p = x^2 - 1$$

$$y = c_1 \cos x + c_2 \sin x + x^2 - 1$$

$$y(0) = 5 = c_1 - 1$$

$$6 = c_1$$

$$y(\pi) = c_1 \cos \pi + c_2 \sin \pi = 0$$

$$6 \cos \pi + c_2 \sin \pi = 0$$

$$c_2 \sin \pi = -6 \cos \pi$$

$$c_2 = \frac{-6 \cos \pi}{\sin \pi}$$

$$= -6 \cot \pi$$

$$y = 6 \cos x - 6(\cot \pi) \sin x + x^2 - 1$$