5.3#1 $W = 16$ lb stretches spring $\frac{8}{3}$ ft.

$x'(0) = 0, \quad x(0) = 2$

$\beta = \frac{1}{2} \qquad f(t) = 10\cos 3t$

$mx'' + \beta x' + kx = f(t)$

$\frac{1}{2}x'' + \frac{1}{2}x' + 6x = f(t)$

$x'' + x' + 12x = 2f(t)$

$x'' + x' + 12x = 20\cos 3t$

$16 = \frac{8}{3}k$

$\frac{3 \cdot 16}{8} = k$

$6 = k$

$16 = mg$

$16 = m \cdot 32$

$\frac{1}{2} = m$

$m = \frac{-1 \pm \sqrt{1-48}}{2}$

$= \frac{-1 \pm i\sqrt{47}}{2}$

$x_c = e^{-\frac{t}{2}}\left(C_1 \cos\frac{\sqrt{47}}{2}t + C_2 \sin\frac{\sqrt{47}}{2}t\right)$

$x_p = A\cos 3t + B\sin 3t$

$x_p' = -3A\sin 3t + 3B\cos 3t$

$x_p'' = -9A\cos 3t - 9B\sin 3t$

$-9A\cos 3t - 9B\sin 3t - 3A\sin 3t + 3B\cos 3t + 12A\cos 3t + 12B\sin 3t = 20\cos 3t$

$-9A + 3B + 12A = 20, \quad -9B - 3A + 12A = 0$

$3B + 3A = 20 \qquad -9B + 9A = 0$

$A = B$

$\Rightarrow 6A = 20$

$A = \frac{20}{6} = \frac{10}{3} = B$

$x = e^{-t/2}\left(C_1 \cos\frac{\sqrt{47}}{2}t + C_2 \sin\frac{\sqrt{47}}{2}t\right) + \frac{10}{3}\cos 3t + \frac{10}{3}\sin 3t$

5.3 #1 CONT.

$$X(0) = 2 = c_1 + \frac{10}{3}$$

$$c_1 = 2 - \frac{10}{3} = -\frac{4}{3}$$

$$x' = -\frac{1}{2} e^{-t/2}\left(c_1 \cos\frac{\sqrt{47}}{2}t + c_2 \sin\frac{\sqrt{47}}{2}t\right) + \frac{\sqrt{47}}{2} e^{-t/2}\left(c_2 \cos\frac{\sqrt{47}}{2}t - c_1 \sin\frac{\sqrt{47}}{2}t\right) - 10\sin 3t + 10\cos 3t$$

$$x'(0) = 0 = -\frac{1}{2}c_1 + \frac{\sqrt{47}}{2}c_2 + 10$$

$$-10 = -\frac{1}{2}\left(-\frac{4}{3}\right) + \frac{\sqrt{47}}{2}c_2$$

$$-10 - \frac{2}{3} = \frac{\sqrt{47}}{2}c_2$$

$$-\frac{32}{3}\cdot\frac{2}{\sqrt{47}} = c_2$$

$$-\frac{64\sqrt{47}}{3\cdot 47} = c_2$$

$$-\frac{64\sqrt{47}}{141} = c_2$$

$$X = e^{-t/2}\left(-\frac{4}{3}\cos\frac{\sqrt{47}}{2}t - \frac{64\sqrt{47}}{141}\sin\frac{\sqrt{47}}{2}t\right) + \frac{10}{3}\cos 3t + \frac{10}{3}\sin 3t$$

5.3 #3 $m = 1$ slug Stretches spring 2 ft. and then comes to rest in the equilibrium position.

$W = mg = 1 \cdot 32$ $32 = 2k$, $16 = k$

$$x'' + 8x' + 16x = 8\sin 4t$$

$x(0) = 0$, $x'(0) = 0$

$$m^2 + 8m + 16 = 0$$
$$(m+4)^2 = 0$$
$$x_c = c_1 e^{-4t} + c_2 t e^{-4t}$$

$$x_p = A\cos 4t + B\sin 4t$$
$$x_p' = -4A\sin 4t + 4B\cos 4t$$
$$x_p'' = -16A\cos 4t - 16B\sin 4t$$

$-16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = 8\sin 4t$

$32B = 0$ $-32A = 8$

$B = 0$ $A = -\frac{1}{4}$

$$x_p = -\frac{1}{4}\cos 4t$$
$$x = c_1 e^{-4t} + c_2 t e^{-4t} - \frac{1}{4}\cos 4t$$
$$x(0) = 0 \Rightarrow c_1 - \frac{1}{4} = 0$$
$$c_1 = \frac{1}{4}$$
$$x'(t) = -4c_1 e^{-4t} - 4c_2 t e^{-4t} + c_2 e^{-4t} + \sin 4t$$
$$x'(0) = 0 \Rightarrow -4c_1 + c_2 = 0$$
$$c_2 = 4c_1 = 1$$
$$x = \frac{1}{4}e^{-4t} + te^{-4t} - \frac{1}{4}\cos 4t$$

5.3 #5 $m = 2$ K $k = 32$ N/m

$x(0) = 0$, $x'(0) = 0$

$f(t) = 68e^{-2t}\cos 4t$

(Notice No Damping)

$mx'' + Bx' + kx = f(t)$

$2x'' + 32x = 68e^{-2t}\cos 4t$

$x'' + 16x = 34e^{-2t}\cos 4t$

$m^2 + 16 = 0$

$m = \pm 4i$

$x_c = C_1\cos 4t + C_2 \sin 4t$

$x_p = Ae^{-2t}\cos 4t + Be^{-2t}\sin 4t =$

$= e^{-2t}(A\cos 4t + B\sin 4t)$

$x_p' = -2e^{-2t}(A\cos 4t + B\sin 4t) + e^{-2t}(-4A\sin 4t + 4B\cos 4t)$

$= e^{-2t}((4B - 2A)\cos 4t - (2B + 4A)\sin 4t)$

$x_p'' = -2e^{-2t}((4B - 2A)\cos 4t - (2B + 4A)\sin 4t)$

$+ e^{-2t}((8A - 16B)\sin 4t - (8B + 16A)\cos 4t)$

$= e^{-2t}((4A - 8B)\cos 4t + (4B + 8A)\sin 4t + (8A - 16B)\sin 4t$

$- (8B + 16A)\cos 4t)$

$= e^{-2t}((-16B - 12A)\cos 4t + (-12B + 16A)\sin 4t)$

$x'' + 16x = 34e^{-2t}\cos 4t$

$e^{-2t}((-16B - 12A)\cos 4t + (16A - 12B)\sin 4t) + 16Ae^{-2t}\cos 4t$

$+ 16Be^{-2t}\sin 4t = 34e^{-2t}\cos 4t$

$-16B - 12A + 16A = 34$, $16A - 12B + 16B = 0$

$-16B + 4A = 34$ $16A + 4B = 0$

$4A = -B$

$-16B - B = 34$

$-17B = 34$

$B = -2$ $\longrightarrow$ $A = -\frac{B}{4} = \frac{2}{4} = \frac{1}{2}$

Mead

5-3 #5 Cont. $x = C_1 \cos 4t + C_2 \sin 4t + \frac{1}{2} e^{-2t} \cos 4t - 2e^{-2t} \sin 4t$

$x(0) = 0 = C_1 + \frac{1}{2}$ $\quad x'(0) = 0$

$C_1 = -\frac{1}{2}$

$x' = -4C_1 \sin 4t + 4C_2 \cos 4t - e^{-2t}(\cos 4t) - 2e^{-2t} \sin 4t$

$+ 4e^{-2t} \sin 4t - 8e^{-2t} \cos 4t$

$x'(0) = 0 = 4C_2 - 1 - 8$

$4C_2 = 9$

$C_2 = \frac{9}{4}$

$$\boxed{x = -\frac{1}{2} \cos 4t + \frac{9}{4} \sin 4t + e^{-2t}\left(\frac{1}{2} \cos 4t - 2 \sin 4t\right)}$$

5.3#7 $mx'' + \beta x' + kx = h(t)$

$x(0)=0, \quad x'(0)=0$

5.3#9 $m = 100\,g \qquad k = 1600$

5.3#13 $m = \frac{1}{2}\,slug \cdot \quad k = 6 \qquad \beta = 2$

$f(t) = 40\sin 2t$

$x(0)=0, \quad x'(0)=0$?

$\frac{1}{2}x'' + 2x' + 6x = 40\sin 2t$

$x'' + 4x' + 12x = 80\sin 2t$

$m^2 + 4m + 12 = 0$

$m = \frac{-4 \pm \sqrt{16-48}}{2}$

$= \frac{-4 \pm \sqrt{-32}}{2}$

$= \frac{-4 \pm i4\sqrt{2}}{2}$

$= -2 \pm 2i\sqrt{2} \rightarrow$ underdamped

$x_p = A\cos 2t + B\sin 2t$

$x_p' = -2A\sin 2t + 2B\cos 2t$

$x_p'' = -4A\cos 2t - 4B\sin 2t$

$-4A\cos 2t - 4B\sin 2t + 8B\cos 2t - 8A\sin 2t + 12A\cos 2t + 12B\sin 2t = 80\sin 2t$

$-4A + 8B + 12A = 0 \qquad -4B - 8A + 12B = 80$

$8A + 8B = 0 \qquad -8A + 8B = 80$

$A = -B \qquad -8A - 8A = 80$

$-16A = 80$

$A = -5$

$B = 5$

$x_p = -5\cos 2t + 5\sin 2t$

5.3#13 Cont.

$$-5\cos 2t + 5\sin 2t = A\sin(2t+\phi)$$

$$\frac{-5}{A} = \sin\phi \qquad \frac{5}{A} = \cos\phi \qquad A = \sqrt{5^2+5^2} = \sqrt{50} = 5\sqrt{2}$$

$$\phi = \tan^{-1}\frac{-5}{5} = \tan^{-1}(-1)$$

$$\phi = \frac{-\pi}{4}$$

$$x_s = 5\sqrt{2}\sin\left(2t - \frac{\pi}{4}\right)$$

5.3#15 a) $x'' + \omega^2 x = F_0\cos\gamma t \qquad x(0)=0,\ x'(0)=0$

$$m^2 + \omega^2 = 0$$

$$m = \pm i\omega$$

$$x_c = c_1\cos\omega t + c_2\sin\omega t$$

$$x_p = A\cos\gamma t + B\sin\gamma t$$

$$x_p' = -A\gamma\sin\gamma t + B\gamma\cos\gamma t$$

$$x_p'' = -A\gamma^2\cos\gamma t - B\gamma^2\sin\gamma t$$

$$-A\gamma^2\cos\gamma t - B\gamma^2\sin\gamma t + \omega^2 A\cos\gamma t + \omega^2 B\sin\gamma t = F_0\cos\gamma t$$

$$-B\gamma^2 + \omega^2 B = 0, \qquad -A\gamma^2 + \omega^2 A = F_0$$

$$B = 0 \qquad A(\omega^2 - \gamma^2) = F_0$$

$$A = \frac{F_0}{\omega^2-\gamma^2}$$

$$x_p = \frac{F_0}{\omega^2-\gamma^2}\cos\gamma t$$

$$x = c_1\cos\omega t + c_2\sin\omega t + \frac{F_0}{\omega^2-\gamma^2}\cos\gamma t$$

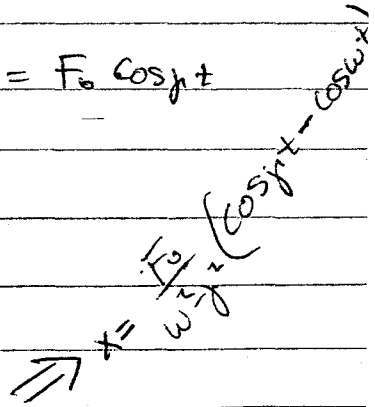
$$x(0) = 0 = c_1 + \frac{F_0}{\omega^2-\gamma^2} \Rightarrow c_1 = \frac{-F_0}{\omega^2-\gamma^2}$$

$$x' = -c_1\omega\sin\omega t + c_2\omega\cos\omega t - \frac{F_0\gamma}{\omega^2-\gamma^2}\sin\gamma t$$

$$x'(0) = 0 = c_2\omega$$

$$c_2 = 0$$

mod



5.3 #15b. Evaluate $\lim_{\gamma \to \omega} \frac{F_0}{\omega^2 - \gamma^2} (\cos \gamma t - \cos \omega t)$

$$= \lim_{\gamma \to \omega} \frac{-F_0 t \sin \gamma t + 0}{-2\gamma}$$

$$= \frac{-F_0 t \sin \omega t}{-2\omega}$$

$$= \frac{F_0 t \sin \omega t}{2\omega}$$

5.3#17

$$\frac{d^2x}{dt^2} + 4x = -5\sin 2t + 3\cos 2t, \quad x(0) = -1, \quad x'(0) = 1$$

$$x_c = C_1 \cos 2t + C_2 \sin 2t$$

$$x_p = At\cos 2t + Bt\sin 2t$$

$$x_p' = -2At\sin 2t + A\cos 2t + 2Bt\cos 2t + B\sin 2t$$
$$= (-2At + B)\sin 2t + (2Bt + A)\cos 2t$$

$$x_p'' = -2A\sin 2t + (-4At + 2B)\cos 2t + 2B\cos 2t - (4Bt + 2A)\sin 2t$$
$$= (-4Bt - 4A)\sin 2t + (-4At + 4B)\cos 2t$$

$$(-4Bt - 4A)\sin 2t + (-4At + 4B)\cos 2t + 4At\cos 2t + 4Bt\sin 2t =$$
$$-5\sin 2t + 3\cos 3t$$

$$-4A = -5, \quad 4B = 3$$
$$A = \frac{5}{4}, \quad B = \frac{3}{4}$$

$$x = C_1\cos 2t + C_2\sin 2t + \frac{5}{4}t\cos 2t + \frac{3}{4}t\sin 2t$$

$$x(0) = -1 = C_1$$

$$x' = -2C_1\sin 2t + 2C_2\cos 2t + \frac{5}{4}\cos 2t - \frac{5}{2}t\sin 2t + \frac{3}{4}\sin 2t + \frac{3}{2}t\cos 2t$$

$$x'(0) = 1 = 2C_2 + \frac{5}{4}$$

$$1 - \frac{5}{4} = 2C_2$$

$$-\frac{1}{4} = 2C_2$$

$$-\frac{1}{8} = C_2$$

$$x = -\cos 2t - \frac{1}{8}\sin 2t + \frac{5}{4}t\cos 2t + \frac{3}{4}t\sin 2t$$