7.3 #1 $\mathcal{L}\{te^{10t}\} = \dfrac{1}{(s-10)^2}$

7.3 #3 $\mathcal{L}\{t^3 e^{-2t}\} = \dfrac{3!}{(s+2)^4}$

7.3 #5 $\mathcal{L}\{e^t \sin 3t\} = \dfrac{3}{(s-1)^2+9}$

7.3 #7 $\mathcal{L}\{e^{5t}\sinh 3t\} = \dfrac{3}{(s-5)^2-9}$

7.3 #9 $\mathcal{L}\{t(e^t+e^{2t})^2\} = \mathcal{L}\{t(e^{2t}+2e^{3t}+e^{4t})\}$

$= \mathcal{L}\{te^{2t}+2te^{3t}+te^{4t}\}$

$= \dfrac{1}{(s-2)^2} + \dfrac{2}{(s-3)^2} + \dfrac{1}{(s-4)^2}$

7.3 #11 $\mathcal{L}\{e^{-t}\sin^2 t\} = \mathcal{L}\left\{e^{-t}\dfrac{(1-\cos 2t)}{2}\right\}$

$= \mathcal{L}\left\{\dfrac{e^{-t}}{2} - \dfrac{e^{-t}\cos 2t}{2}\right\}$

$= \dfrac{1}{2(s+1)} - \dfrac{1}{2}\cdot\dfrac{s+1}{(s+1)^2+4}$

7.3 #13 $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\} = \dfrac{e^{-2t}t^2}{2}$

7.3 #15 $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-6s+10}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s^2-6s+9+1}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{1}{(s-3)^2+1}\right\}$

$= e^{3t}\sin t$

7.3 #17 $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+4+1}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s+2-2}{(s+2)^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+1}\right\}$

$= e^{-2t}\cos t - 2e^{-2t}\sin t$

7-3 # 19 $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s+1} + \frac{B}{(s+1)^2}\right\}$

where $A(s+1) + B = s$

$As = s \quad A+B=0$

$A=1 \quad B=-1$

$= \mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{1}{(s+1)^2}\right\}$

$= e^{-t} - te^{-t}$

7.8 #21 $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\}$

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)^3} = \frac{2s-1}{s^2(s+1)^3}$

$As(s+1)^3 + B(s+1)^3 + Cs^2(s+1)^2 + Ds^2(s+1) + Es^2 = 2s-1$

$s=0 \Rightarrow B=-1, \; s=-1 \Rightarrow E=-3$

$s=1 \Rightarrow 8A+8B+4C+2D+E=1$

$8A-8+4C+2D-3=1$

$8A+4C+2D=12$

$\Longrightarrow -8C+4C+2D=12$

$-4C+2D=12$

$As^4+Cs^4=0s^4$

$A+C=0 \Rightarrow A=-C$

$3As^3+Bs^3+2Cs^3+Ds^3=0s^3$

$3A+B+2C+D=0$

$-C+B+D=0$

$-C+D=1$

$D=1+C$

7.3 #21 continued

$$-4c+2(1+c)=12$$
$$-4c+2+2c=12$$
$$-2c=10$$
$$c=-5$$

$$D=-4$$
$$A=5$$
$$B=-1$$
$$E=-3$$

$$\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\}=\mathcal{L}^{-1}\left\{\frac{5}{s}-\frac{1}{s^2}-\frac{5}{(s+1)}-\frac{4}{(s+1)^2}-\frac{3}{(s+1)^3}\right\}$$
$$=5-t-5e^{-t}-4te^{-t}-\frac{3}{2}t^2e^{-t}$$

7.3 #23 $\mathcal{L}\{(t-1)\,\mathcal{U}(t-1)\}=\frac{e^{-s}}{s^2}$

7.3 #25 $\mathcal{L}\{t\,\mathcal{U}(t-2)\}=\mathcal{L}\{(t-2)\,\mathcal{U}(t-2)+2\mathcal{U}(t-2)\}$
$$=\frac{e^{-2s}}{s^2}+\frac{2e^{-2s}}{s}$$

7.3 #27 $\mathcal{L}\{\cos 2t\,\mathcal{U}(t-\pi)\}=\mathcal{L}\{\cos 2(t-\pi)\,\mathcal{U}(t-\pi)\}$
$$=\frac{se^{-\pi s}}{s^2+4}$$

7.3 #29 $\mathcal{L}\{(t-1)^3e^{t-1}\,\mathcal{U}(t-1)\}=$
$$=e^{-s}\mathcal{L}\{t^3e^t\}$$
$$=e^{-s}\cdot\frac{3!}{(s-1)^4}$$

7.3 # 31 $\quad \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\} = \frac{1}{2}(t-2)^2\mathcal{U}(t-2)$

7.3 # 33 $\quad \mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} = \sin(t-\pi)\,\mathcal{U}(t-\pi)$

7.3 # 35 $\quad \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{e^{-s}\cdot\left(\frac{A}{s}+\frac{B}{s+1}\right)\right\}$ NO

↓ Better

$$\mathcal{L}^{-1}\left\{e^{-s}\cdot\frac{s+1-s}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{e^{-s}\left(\frac{s+1}{s(s+1)} - \frac{s}{s(s+1)}\right)\right\}$$

$$= \mathcal{L}^{-1}\left\{e^{-s}\cdot\left(\frac{1}{s} - \frac{1}{s+1}\right)\right\}$$

$$= \mathcal{U}(t-1) - e^{-(t-1)}\mathcal{U}(t-1)$$

7.3 # 37 $\quad \mathcal{L}\{t\cos 2t\} = -\frac{d}{ds}\mathcal{L}\{\cos 2t\}$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+4}\right)$$

$$= -\left[\frac{s^2+4-2s^2}{(s^2+4)^2}\right]$$

$$= \frac{s^2-4}{(s^2+4)^2}$$

7.3 #39 $\mathcal{L}\{t^2 \sinh t\} = \frac{d^2}{ds^2}\mathcal{L}\{\sinh t\}$

$$= \frac{d^2}{ds^2}\left[\frac{1}{s^2-1}\right]$$

$$= \frac{d}{ds}\left[\frac{-2s}{(s^2-1)^2}\right]$$

$$= \frac{-2(s^2-1)^2+8s^2(s^2-1)}{(s^2-1)^4}$$

$$= \frac{(s^2-1)(-2(s^2-1)+8s^2)}{(s^2-1)^4}$$

$$= \frac{-2s^2+2+8s^2}{(s^2-1)^3}$$

$$= \frac{6s^2+2}{(s^2-1)^3}$$

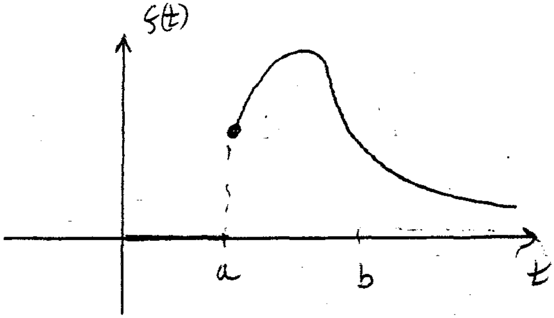
7.3 #41 $\mathcal{L}\{te^{2t}\sin 6t\} = -\frac{d}{ds}\left[\frac{6}{(s-2)^2+36}\right]$

$$= \frac{12(s-2)}{((s-2)^2+36)^2}$$

7.3 #43 $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \mathcal{L}^{-1}\left\{-\frac{1}{2}\cdot\frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right\}$

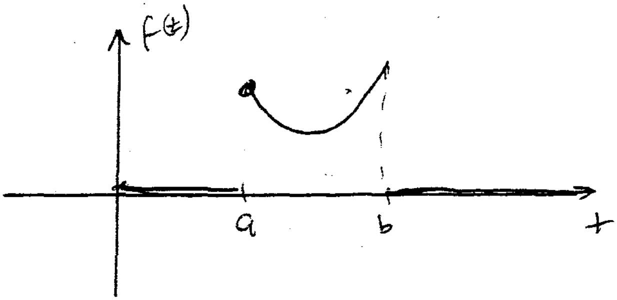
$$= \frac{1}{2}t\sin t$$

7.3#45



$f(t)\,u(t-a)$ (c)

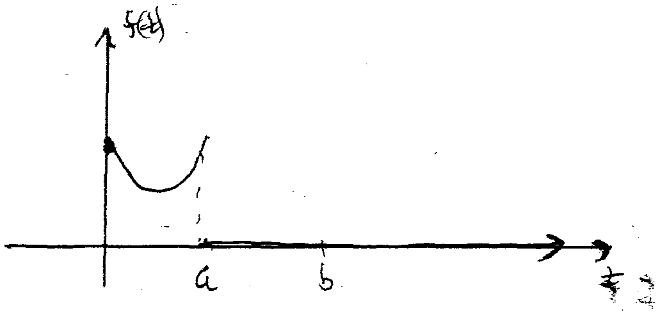
7-3#47



$f(t-a)\,u(t-a) - f(t-a)\,u(t-b)$

(f)

7.3#49



$f(t) - f(t)\,u(t-a)$ (a)

7.3#51 $f(t) = \begin{cases} 2, & 0 \le t < 3 \\ -2 & t \ge 3 \end{cases}$

$$f(t) = 2 - 4u(t-3)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2 - 4u(t-3)\}$$
$$= \frac{2}{s} - \frac{4e^{-3s}}{s}$$

7.3#53 $f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t^2 & t \ge 1 \end{cases}$

$$f(t) = t^2 u(t-1)$$

$$\mathcal{L}\{t^2 u(t-1)\}$$

$$= \mathcal{L}\{(t^2 - 2t + 1)u(t-1) + (2t-1)u(t-1)\}$$

$$= \mathcal{L}\{(t-1)^2 u(t-1) + (t-1)u(t-1) + t\,u(t-1)\}$$

$$= \mathcal{L}\{(t-1)^2 u(t-1) + (t-1)u(t-1) + (t-1)u(t-1) + u(t-1)\}$$

$$= \frac{2e^{-s}}{s^3} + \frac{2e^{-s}}{s^2} + \frac{e^{-s}}{s}$$

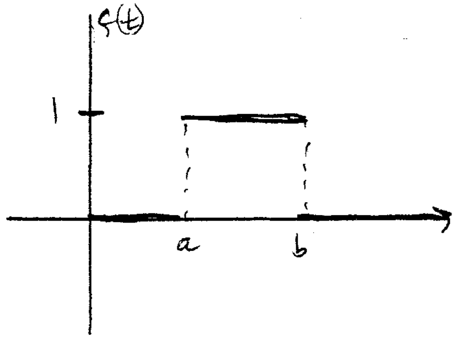
OR $\mathcal{L}\{t^2 u(t-1)\} = \frac{d^2}{ds^2}\mathcal{L}\{u(t-1)\}$

$$= \frac{d^2}{ds^2}\left[\frac{e^{-s}}{s}\right] = \frac{d}{ds}\left[\frac{-se^{-s} - e^{-s}}{s^2}\right]$$

$$= \frac{\left[(-e^{-s} + se^{-s} + e^{-s})s^2 - 2s(-se^{-s} - e^{-s})\right]}{s^4}$$

$$= \frac{2e^{-s}}{s^3} + \frac{2e^{-s}}{s^2} + \frac{e^{-s}}{s} \leftarrow = \frac{\left[s^3 e^{-s} + 2s^2 e^{-s} + 2se^{-s}\right]}{s^4}$$

7.3 # 57



$$f(t) = u(t-a) - u(t-b)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{u(t-a) - u(t-b)\}$$

$$= \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$$

7.3 # 59

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{e^{-s}}{s^2}\right\}$$

$$= t - (t-1)u(t-1)$$