In Problems 1 and 2 use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$.

1. $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & t \ge 1 \end{cases}$

2. $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ 1, & 2 \le t < 4 \\ 0, & t \ge 4 \end{cases}$

Without referring back to the text, fill in the blanks.

3. $\mathcal{L}\{e^{-7t}\} =$ _____

4. $\mathcal{L}\{te^{-7t}\} =$ _____

5. $\mathcal{L}\{\sin 2t\} =$ _____

6. $\mathcal{L}\{e^{-3t}\sin 2t\} =$ _____

7. $\mathcal{L}\{t \sin 2t\} =$ _____

8. $\mathcal{L}\{\sin 2t\,\mathcal{U}(t - \pi)\} =$ _____

9. $\mathcal{L}^{-1}\left\{\dfrac{20}{s^6}\right\} =$ _____

10. $\mathcal{L}^{-1}\left\{\dfrac{1}{4s + 1}\right\} =$ _____

11. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s - 5)^3}\right\} =$ _____

12. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2 - 5}\right\} =$ _____

13. $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2 - 10s + 29}\right\} =$ _____

14. $\mathcal{L}^{-1}\left\{\dfrac{e^{-5s}}{s^2}\right\} =$ _____

In Problems 15–17 use the Laplace transform to solve the given equation.

15. $y'' - 2y' + y = e^t, \quad y(0) = 0, \quad y'(0) = 5$

16. $y'' - 8y' + 20y = te^t, \quad y(0) = 0, \quad y'(0) = 0$

17. $y' - 5y = f(t)$ where $f(t) = \begin{cases} t^2, & 0 \le t < 1, \\ 0, & t \ge 1, \end{cases} \quad y(0) = 1.$

Answers

1. $\frac{1}{s^2} - \frac{2}{s^2}e^{-s}$

2. $\frac{1}{5}\left(e^{-2s} - e^{-4s}\right)$

3. $\frac{1}{s+7}$

4. $\frac{1}{(s+7)^2}$

5. $\frac{2}{s^2+4}$

6. $\frac{2}{(s+3)^2+4}$

7. $\frac{4s}{(s^2+4)^2}$

8. $\frac{2e^{-\pi s}}{s^2+4}$

9. $\frac{t^5}{6}$

10. $\frac{1}{4}e^{-1/4\,t}$

11. $\frac{1}{2}t^2e^{5t}$

12. $\frac{\sinh\sqrt{5}\,t}{\sqrt{5}}$

13. $e^{5t}\left(\cos 2t + \frac{5}{2}\sin 2t\right)$

14. $\mathcal{U}(t-5)(t-5)$

15. $y = 5te^t + \frac{1}{2}t^2e^t$

17. $y = -\frac{2}{125} - \frac{2}{25}t - \frac{1}{5}t^2 + \frac{127}{125}e^{5t}$
$-\left[-\frac{37}{125} - \frac{12}{25}(t-1) - \frac{1}{5}(t-1)^2 + \frac{37}{125}e^{5(t-1)}\right]\mathcal{U}(t-1)$

16. $y = \frac{6}{169}e^t + \frac{1}{13}te^t - \frac{6}{169}e^{4t}\cos 2t + \frac{5}{338}e^{4t}\sin 2t$